PHY905 Section 4
Ion Accelerator Technology
Lecture 5
-Superconductivity Fundamentals for SRF-

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Syllabus: Superconducting Radiowave Frequency Basics

My lecture will teach SRF basics for students without SRF cavity experience, and will concentrate mostly on SRF cavities and includes a lot of technics related to cavity fabrication:

1)	Superconductivity for SRF technology (to understand the SRF cavities),	Jan 24
2)	High purity niobium production (for SRF cavities required high performance),	Jan 26
3)	SRF cavity design,	Jan 31
4)	SRF cavity fabrication and cavity cold test method,	Feb 02
5)	Performance limitations of SRF cavities and mitigations,	Feb 28
6)	Cavity surface treatment techniques,	Mar 02
7)	Hydrogen absorption issue and Clean assembly,	Mar 07
8)	High gradient cavity R&D for ILC,	Mar 09
10)	Cavity accessories: coupler, HOM, tuner, tuner,	Mar 14
11))	Others,	Mar 16

If time is allowed, others: superconducting solenoid, and a very brief explanation for cryogenics will be made.

My lecture **dates**: 2017 1/24, 1/26, 1/31, 2/02, 2/28, 3/02, 3/07, 3/09, 3/14, 3/16

Time: 2:00 – 3:30 PM (Tuesday)

3:30 – 5:00 PM (Thursday)

Location: 1200 Lecture Hall, NSCL/FRIB Bldg. Tower 1

Superconductivity for SRF technology

5.1 Superconductivity (SC) for SRF

5.2 What happens on the SRF surface when microwave exposes

5.3 Important SC material properties for SRF application and the measurement methods

5.4 Abrikosov's theory for Type-II Superconductors

5.5 SC magnetic properties of very high purity lab niobium specimens and industrial produced materials

5.5 Why surface resistance in SRF application

5.6 Fundamental limitation for SRF application

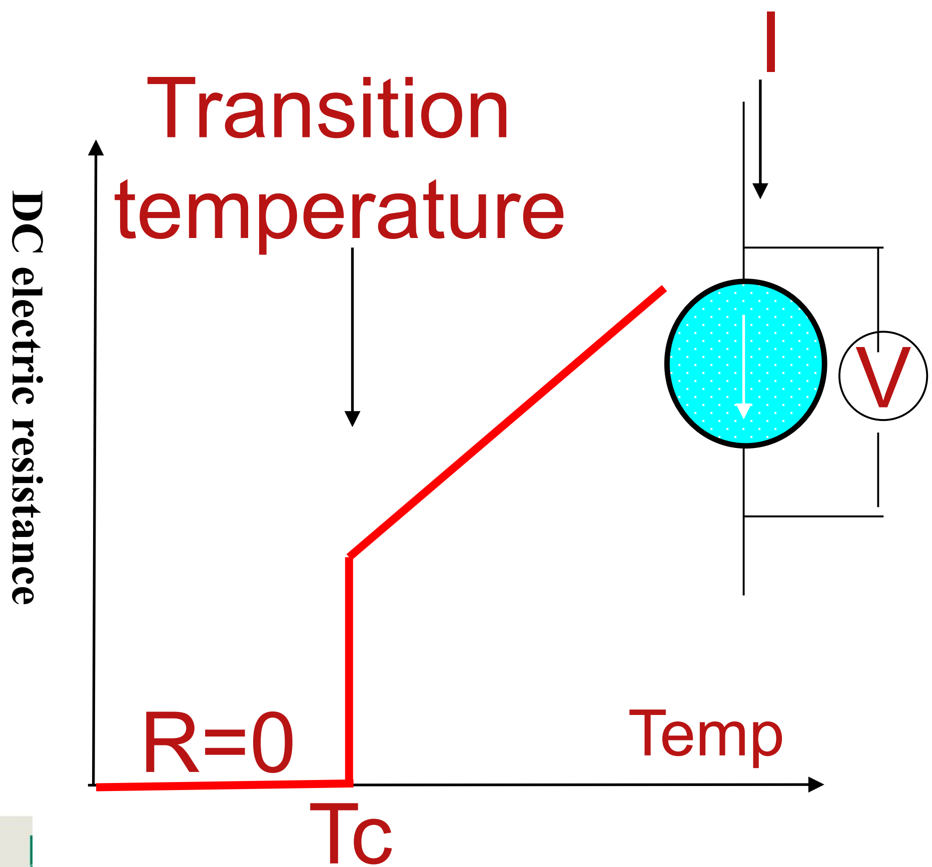
5.1 Superconductivity for SRF

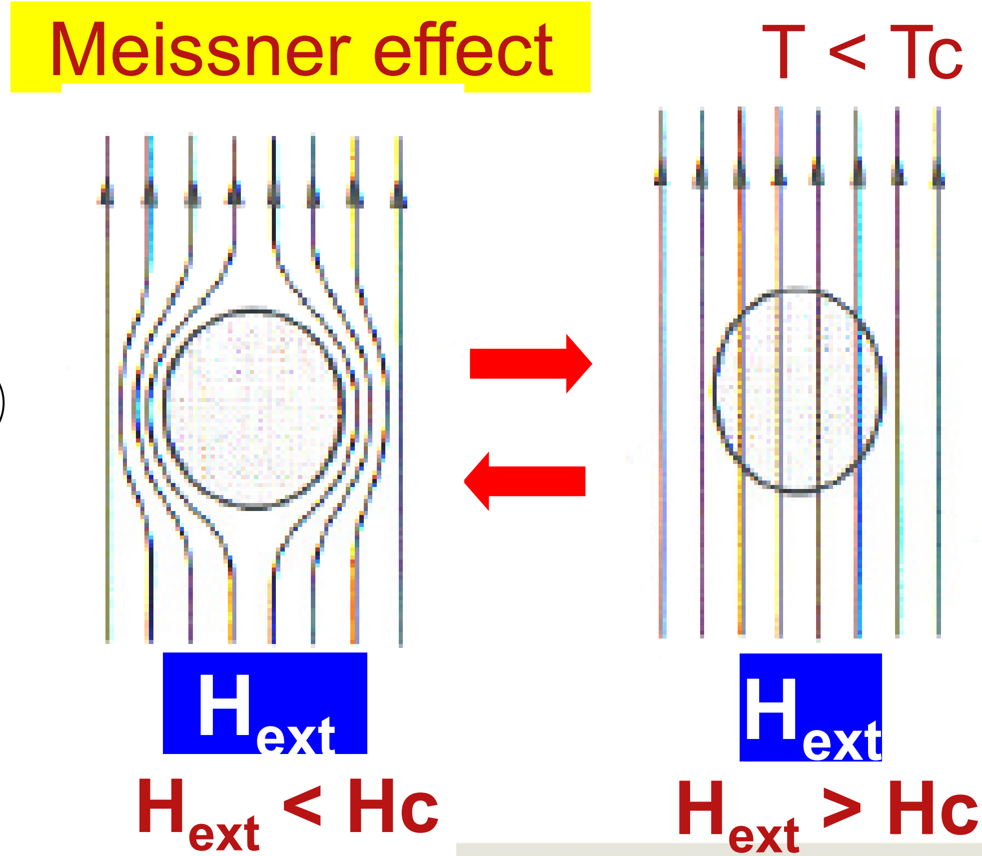
1911 by K. Onnes

19933 by Meissner and Ochsenfeld (experiment)
1935 Phenomenological theory by F. and H. London

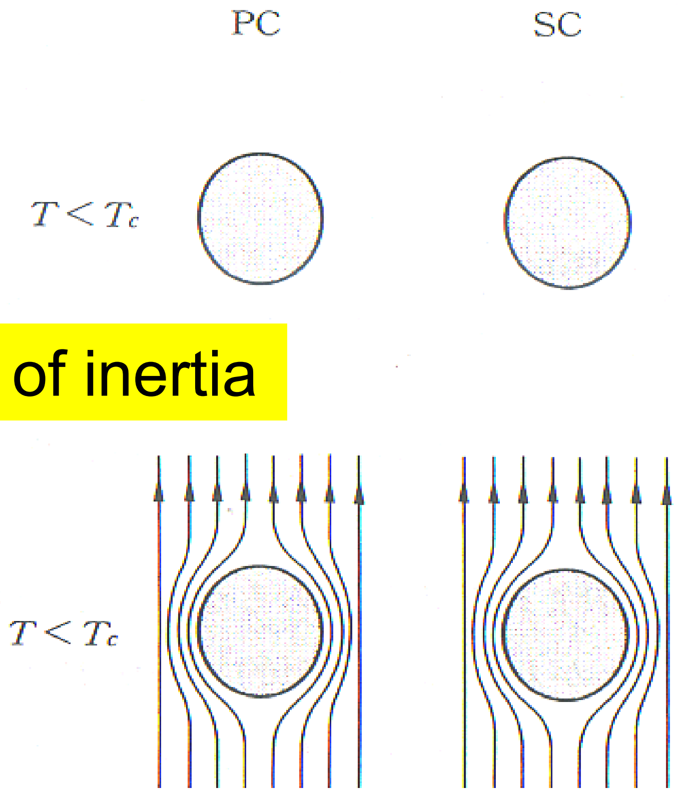
Zero DC resistance at Tc

Perfect diamagnetism < Hc



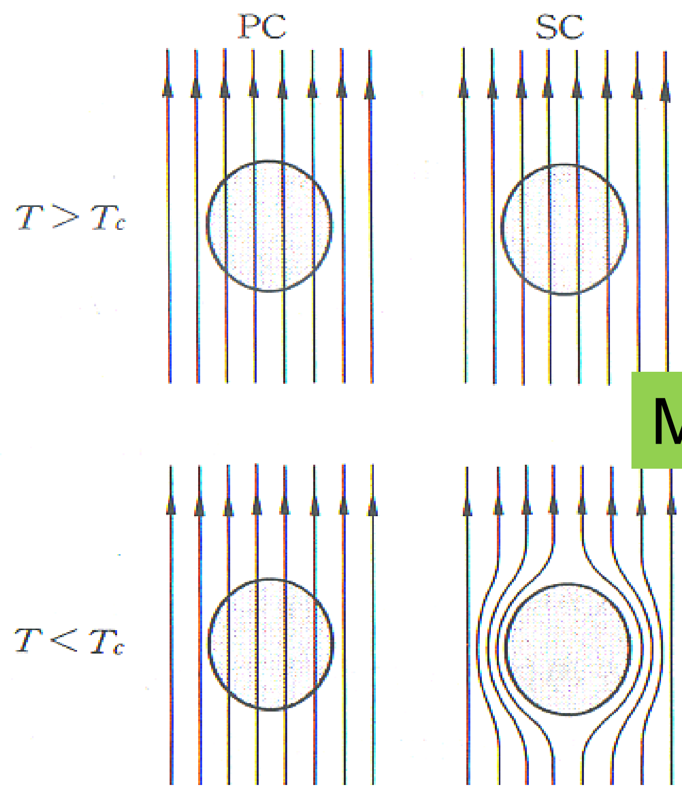


Deference between perfect conductor (PC) and superconductor (SC)



Law of inertia

(a) The case: the specimen is applied external magnetic field at T < Tc

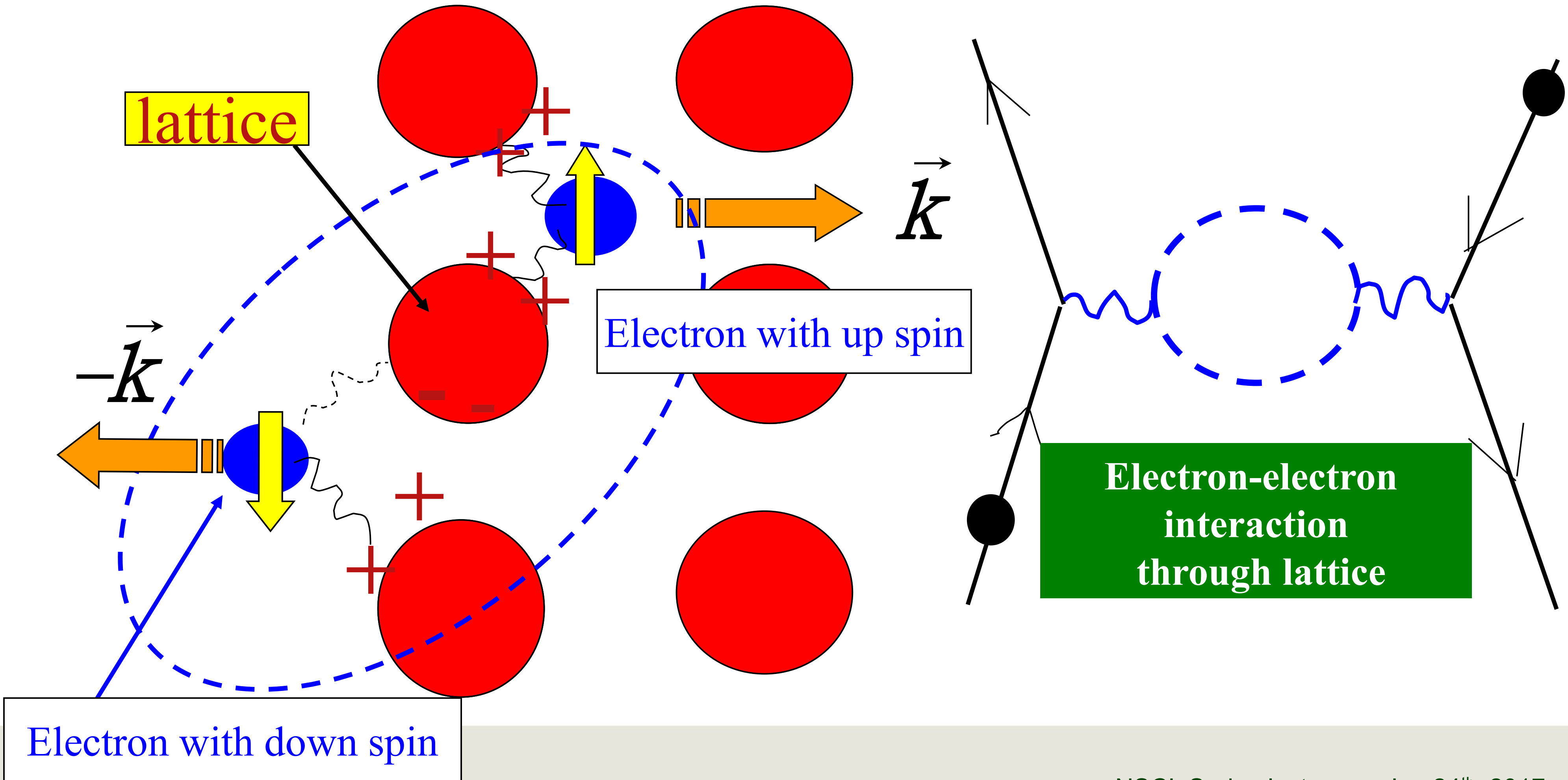


Meissner effect

(b) The case: the specimen is applied external magnetic field at T > Tc and cooled down. PC keeps the field inside by the law of inertia, but SC expels the field out of the material by Meissner effect. The law inertia is the character of the time-space. Meissner effect is the character of materials.

Electron-electron interaction through the lattice in superconducting state

Electron/electron have Coulomb interaction and repel each other, however the interaction through lattices can make attractive force → **Superconducting state**



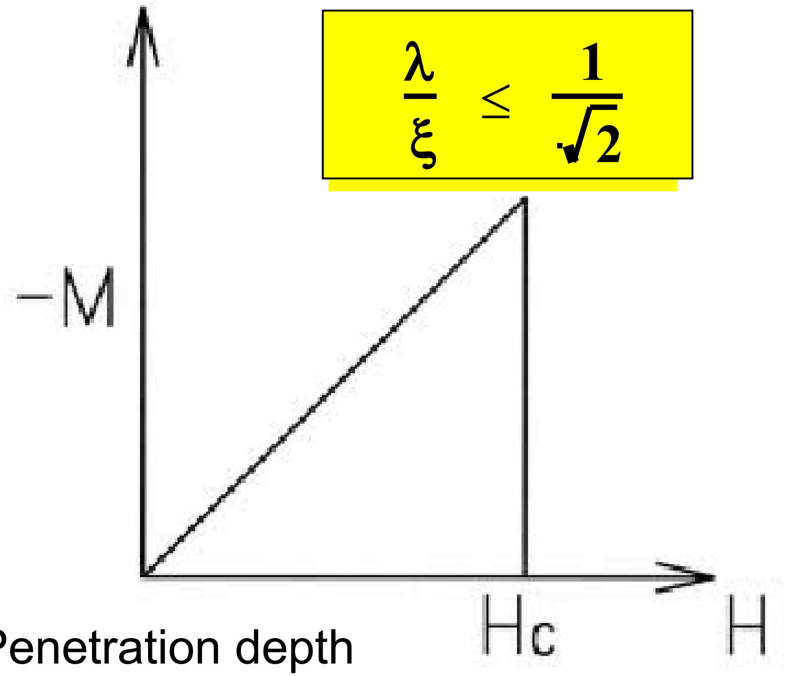
History of Superconductivity

1908 Production of liquid Helium H.K. Onnes
1911 Discovery, H.K Onnes. Persistent current、Critical magnetic field
（**1913　Novel Price：Research of the low temperature property related to production of liquid Helium**)
1933 Exclusion of magnetic field, Meissner and Ochsenfeld
1934 Two fluid model, Gorter and Casimir
1935 Phonological theory (London's penetration depth), F. London and H. London
1937 Discovery of Type-II superconductor, Schubnikov
1950 Phonological theory of superconductivity (GL theory), Ginzburug and Landau
Pointing of Electron・phonon interaction, Fröhlich
Isotope effect on Tc, Maxwell, Reynolds
1953 Concept of Coherent length (non-localization in London equation), Prediction of energy gap, Pippard
Experimental evidence of the energy gap, Goodman and others
1957 BCS theory (microscopic model of superconductivity), Bardeen, Cooper, and Schriefer
(**1972 Novel price for BCS theory**)
Theory for Type-II superconductors, Abrikosov
（**2003 Novel price for Abrikosov's pioneer contribution to superconductivity**)
1960 Spontaneous symmetry breaking in superconductivity, Nanbu, Goldstone, Anderson
1961 Verification of quantum Fluxiod, Fairbank, Näbauer
1962 Discovery of Josenphson effect　Josephson
1986 Discovery of High Tc superconductors (ceramic), Bednorz, Müller

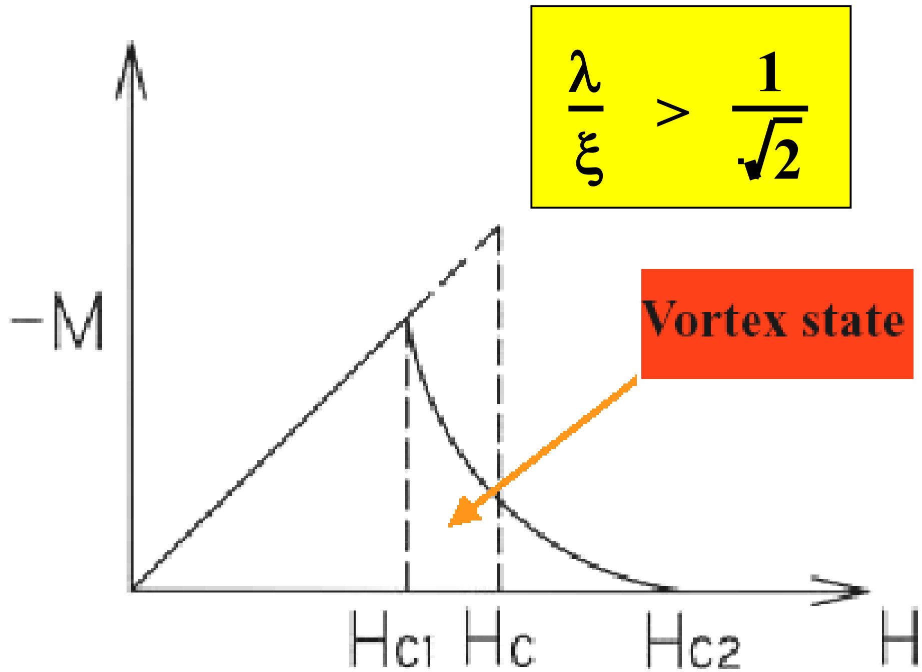
It took a long time (over 50 years) for perfect understating, from these complicate phenomena. Many Novel Prices for superconductivity

Two types of Superconductor

Type-I



Type-II



λ: Penetration depth
ξ: Coherent length
Hc: thermodynamic critical magnetic field
Hc1: Lower critical magnetic field in type-II superconductors
Hc2: Upper critical magnetic field in type-II superconductors

In the early stage, physicist thought type-II superconductor might be low purity material of type-I superconductor and were confused. It's true that high purity Lead is the type-I but the lower purity Lead shoes type-II SC character. Abrikosov has pointed out Type-II superconductors itself exists.

Superconductors (metals)

Table 1 Superconductivity parameters of the elements

An asterisk denotes an element superconducting only in thin films or under high pressure in a crystal modification not normally stable. Data courtesy of B. T. Matthias, revised by T. Geballe.

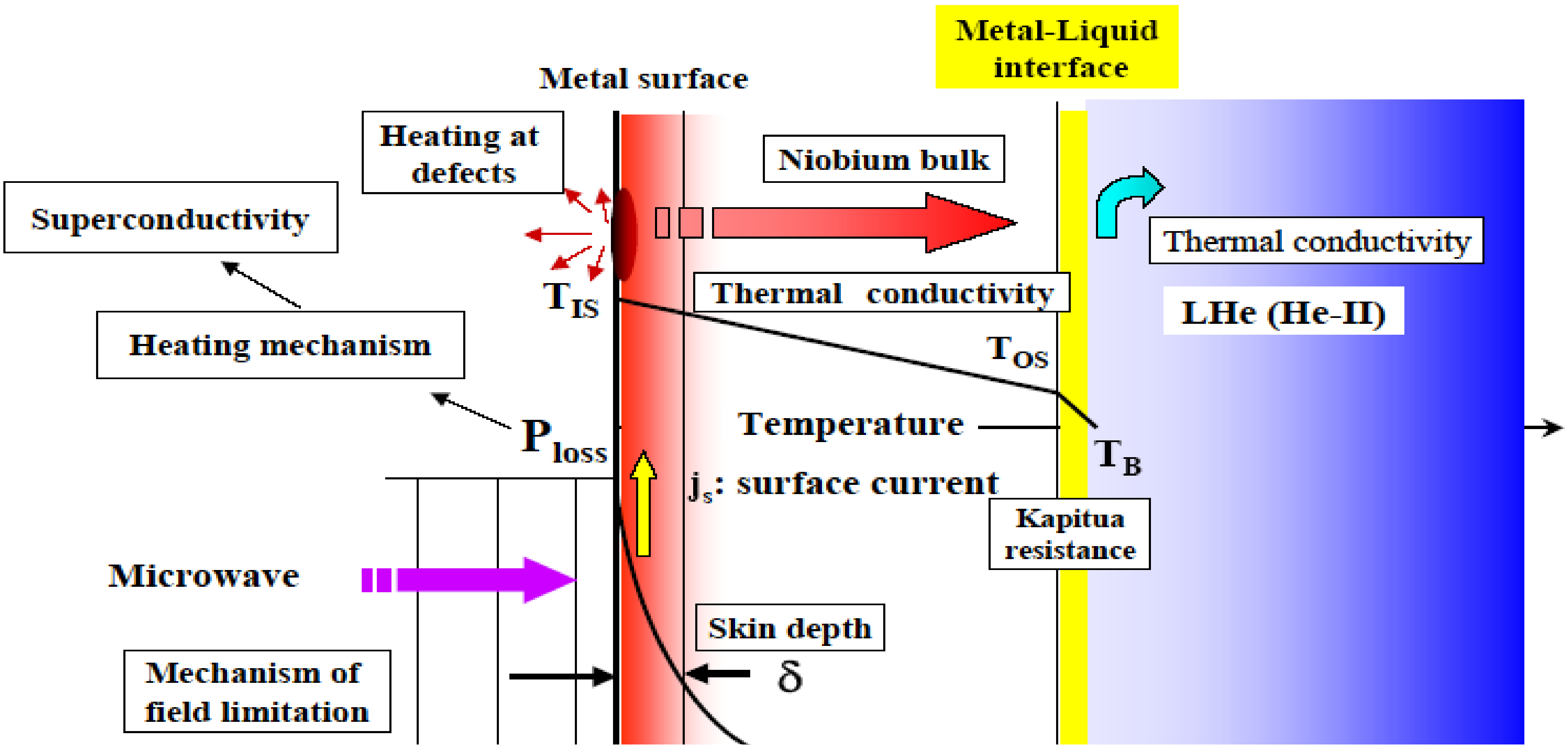
Transition temperature in K
Critical magnetic field at absolute zero in gauss (10^{-4} tesla)

Li	Be 0.026											B	C	N	O	F	Ne
Na	Mg											Al 1.140 105	Si*	P*	S*	Cl	Ar
K	Ca	Sc	Ti 0.39 100	V 5.38 1420	Cr*	Mn	Fe	Co	Ni	Cu	Zn 0.875 53	Ga 1.091 51	Ge*	As*	Se*	Br	Kr
Rb	Sr	Y*	Zr 0.546 47	**Nb** **9.50** **1980**	Mo 0.92 95	Tc 7.77 1410	Ru 0.51 70	Rh .0003 .049	Pd	Ag	Cd 0.56 30	In 3.4035 293	Sn (w) 3.722 309	Sb*	Te*	I	Xe
Cs*	Ba*	La fcc 6.00 1100	Hf 0.12	Ta 4.483 830	W 0.012 1.07	Re 1.4 198	Os 0.655 65	Ir 0.14 19	Pt	Au	Hg (α) 4.153 412	Tl 2.39 171	Pb 7.193 803	Bi*	Po	At	Rn
Fr	Ra	Ac															

Ce*	Pr	Nd	Pm	Sm	Eu	Gd	Tb	Dy	Ho	Er	Tm	Yb	Lu 0.1
Th 1.368 1.62	Pa 1.4	U*(α)	Np	Pu	Am	Cm	Bk	Cf	Es	Fm	Md	No	Lr

Niobium is the most important martial for SRF application

5.2 What happens on SRF surface under microwave exposing



- Microwave penetrates by a skin depth (London's penetration depth) into the cavity inner wall surface.
- Surface current is induced and flows on the surface skin.
- SRF surface is heated by surface resistance. The heat is transferred into the liquid Helium through the cavity wall.
- Even using ideal superconductor, the accelerating gradient (Eacc ~ B) will be limited by the superconducting properties (fundamental limit).
- Real cavity surface has lots of damages or contamination on the SRF surface. They are also heated by the surface current, and bring a quench below the fundamental limitation.
- The thermal conductivity of the cavity wall plays an very important role.
- Kapitza resistance (Oxide film etc.) can disturbs the heat transfer at the outer cavity wall.

Difference between SC and NC

- SRF performance will limited by the fundamental SC properties but NC has no material limitation (you will learn today what SC parameters bring the fundamental limitation).
- NC cavity performance is limited by RF heating due to the large surface resistance.
- Even superconductivity, SRF also has a surface resistance, however of which value is ~ 10^{-6} of NC (you will learn why surface resistance happens in SRF application later in this section).
- SRF performance is very sensitive to the RF surface quality: defects, damages, contamination, cleanliness…. (you will learn later).
- High quality cavity material production and surface preparation are more important technology in SRF compared to NC (you will learn several concrete example in this lecture).

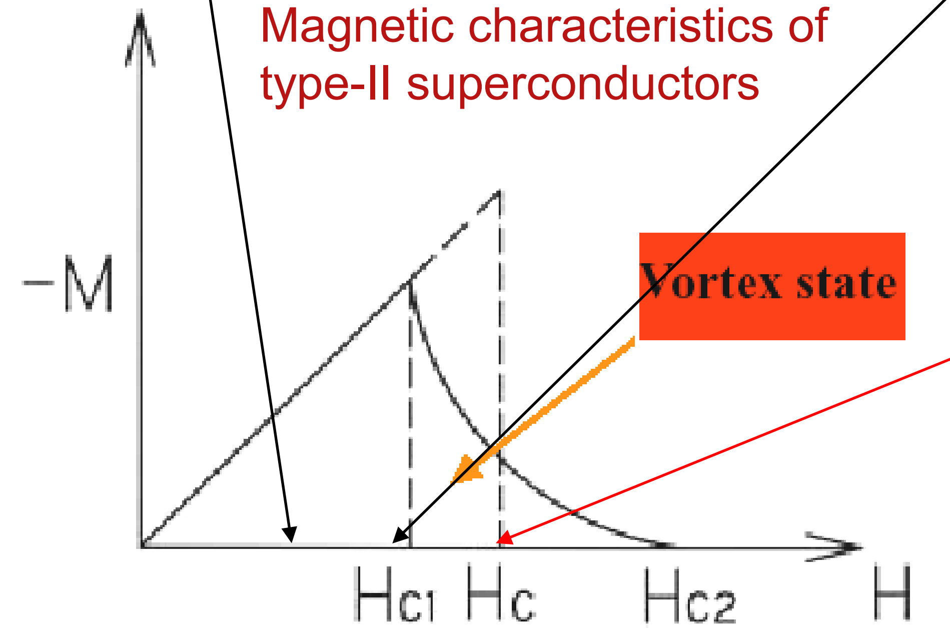
Three causes limited SRF Performance

Surface defects, contamination, dusts…(Engineering)
Thermal conductivity (κ), residual resistance ratio (RRR), Critical temperatures (T_C)

Flux trapping (Technical/fundamental)
Penetration depth (λ), Coherent length (ξ), Lower critical field H_{C1}

Fundamental Limit (SC property)
Critical temperature T_C, Penetration depth (λ), Coherent length (ξ), Thermodynamical critical field (Hc), Upper critical field H_{C2}

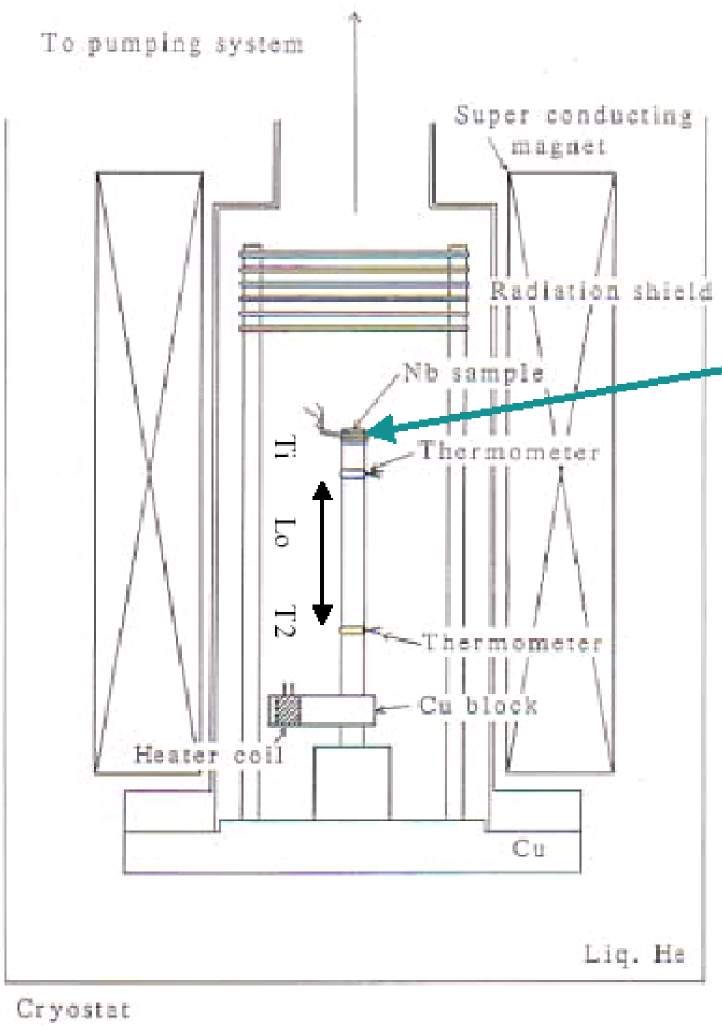
Magnetic characteristics of type-II superconductors



5.3 Important SC Material Parameters and the Measurements

- First, I will explain the important SC parameters for SRF technology in this session.
- The measurement methods and concrete measurement results also introduced, which are really used calculation for SRF fundamental limitation.
- The SC praters:
  - Thermal conductivity (κ)
  - Residual resistance ratio (RRR)
  - Critical temperature (T_C)
  - Low critical magnetic field(H_{C1})
  - Upper critical magnetic field (H_{C2})
  - Thermodynamical magnetic field (H_C)
  - Penetration length (λ)
  - Coherent length (ξ)
  - Ginsburg Landau Parameter (κ)

Thermal conductivity of Superconductors



Normal conductor : $\kappa_{en} = \dfrac{1}{W_{en}} = \left[\dfrac{\rho}{L_O T} + aT^2 \right]^{-1}$

$\rho = \dfrac{\rho_{300K}}{RRR}$ e-impurities scatt.

e-lattices scatt.

Wiedemann-Franz low:

Heater:P[w]

$$\kappa_e = \frac{\pi^2 n k_B^2 \tau}{3m} \cdot T, \quad \frac{\kappa_e}{\sigma} = \frac{\pi^2}{3}\left(\frac{k_B}{e}\right)^2 \cdot T = L_O T$$

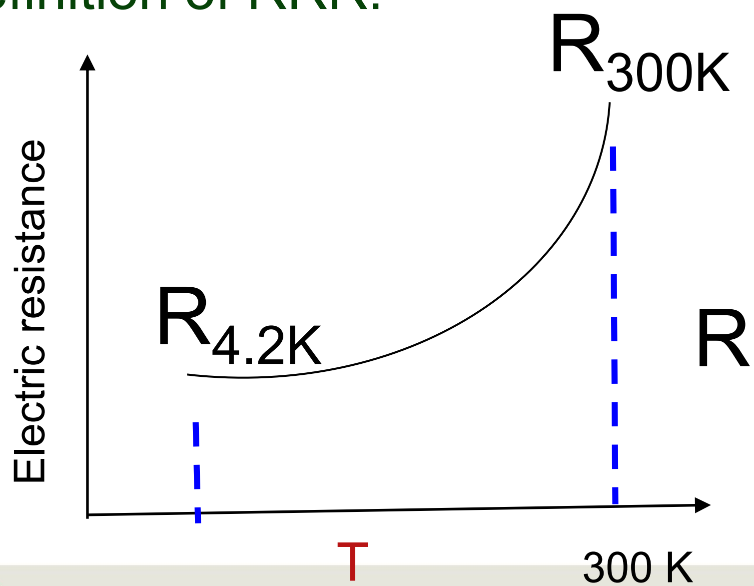
$$P[w] = S(m^2) \cdot \kappa(T) \cdot \frac{T_1(K) - T_2(K)}{L_O(m)}$$

$$T \equiv \frac{T_1 + T_2}{2}, \; S\text{: area of cross-section}$$

$$\kappa(T) = \frac{P}{S} \cdot \frac{L_O}{T_1 - T_2} \qquad [\frac{w}{m \cdot K}]$$

RRR: Residual Resistance Ratio

- Thermal conductivity is a material parameter to evaluate the purity and homogeneity but it is inconvenient to measure many sample simultaneously in the production phase.
- On the other hand RRR is also similar parameter and suitable for multiple sample measurement. SRF community often use RRR instead of thermal conductivity to evaluate niobium material purity.
- RRR is lineally proportional to the thermal conductivity.
- Definition of RRR:

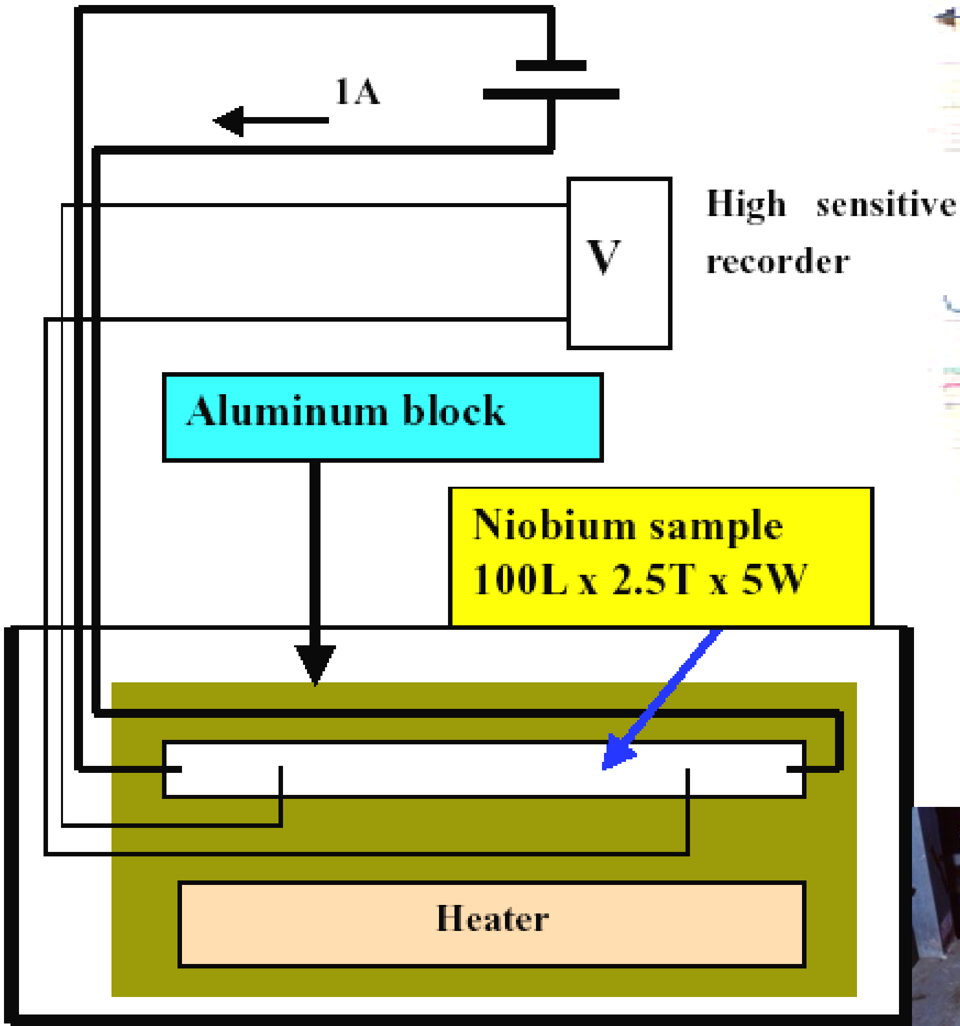


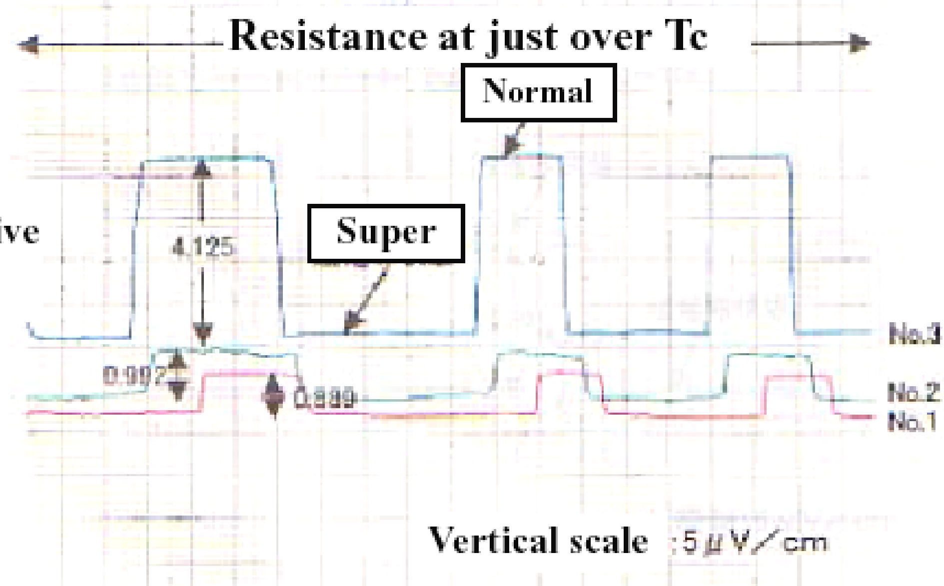
R4.2K brings the information of impurity and inhomogeneity

$$\mathrm{RRR} \equiv \frac{R_{300K}}{R_{4.2K}} \sim \frac{R_{300K}}{R_{9.5K}} \quad \text{for Nb}$$

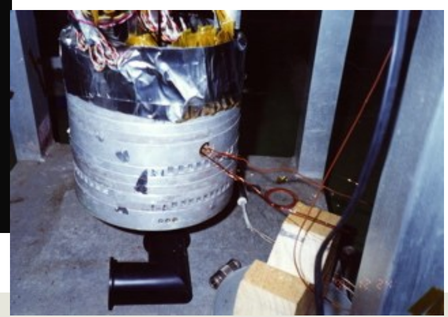
RRR Measurement

- RRR can measure multiple samples at one experiment.
- Very simple measurement method.



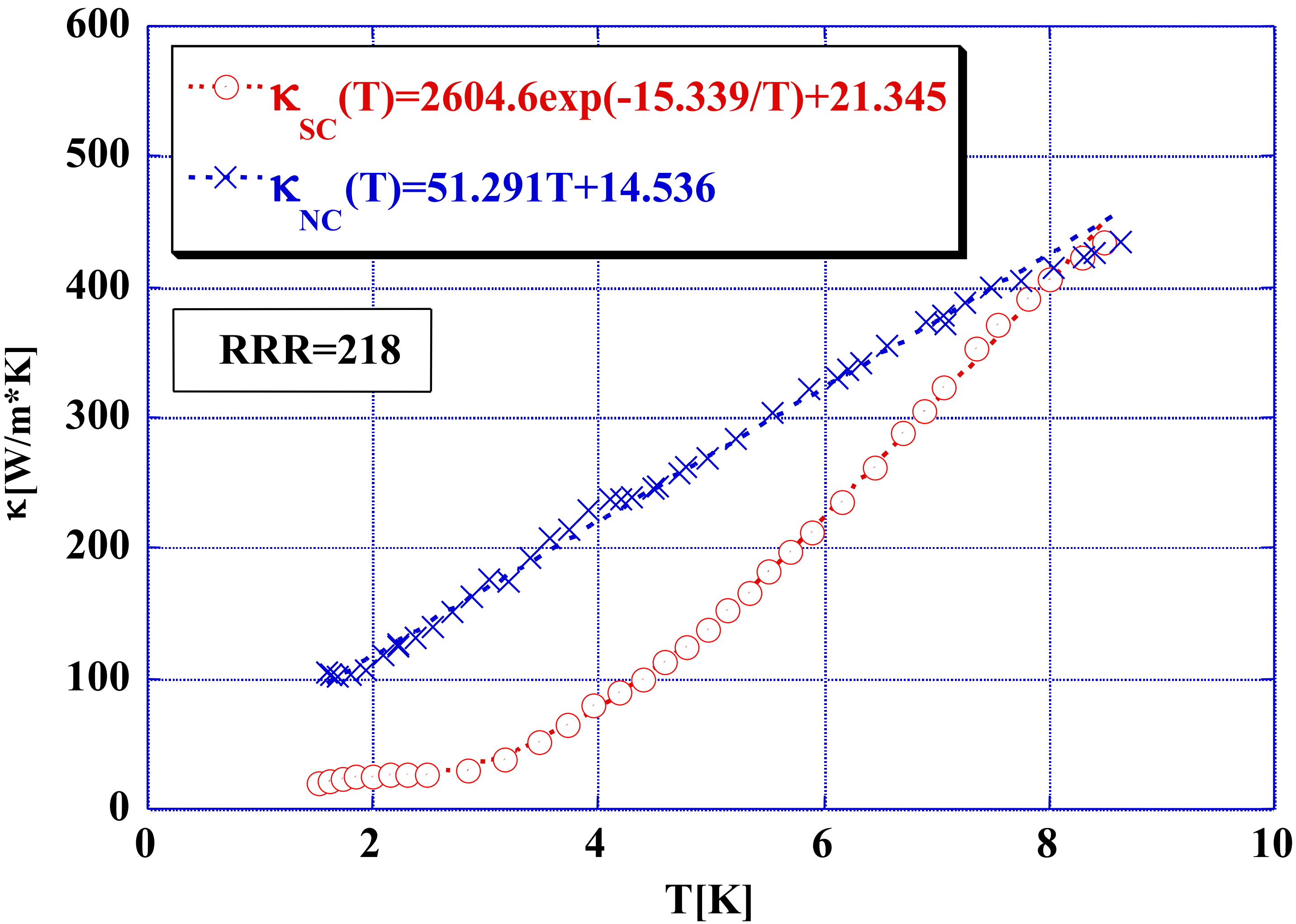


$$\kappa(T) \propto RRR$$



$$RRR \equiv \frac{R(300K)}{R(9.5K)}$$

Thermal conductivity comparison with NC and SC



$$\frac{\Delta}{k_B} = 15.339$$

$$\Downarrow$$

$$\frac{2\Delta}{k_B T_c} = \frac{2 \times 15.339}{k_B\, 9.25}$$

$$2\Delta = 3.317 k_B T_c$$

BCS theory

$$2\Delta = 3.52 k_B T_c$$

Calculation of thermal conductivity based on Quantum mechanics

$$\kappa_s(T) = R(y)\cdot\left[\frac{\rho_{295K}}{L\cdot RRR\cdot T} + a\cdot T^2\right]^{-1} + \left[\frac{1}{D\cdot\exp(y)\cdot T^2} + \frac{1}{BlT^3}\right]^{-1}$$

e- impurities scatt. e- phonons scatt. lattice-phonnons scatt. lattice-grain boundaries scatt.

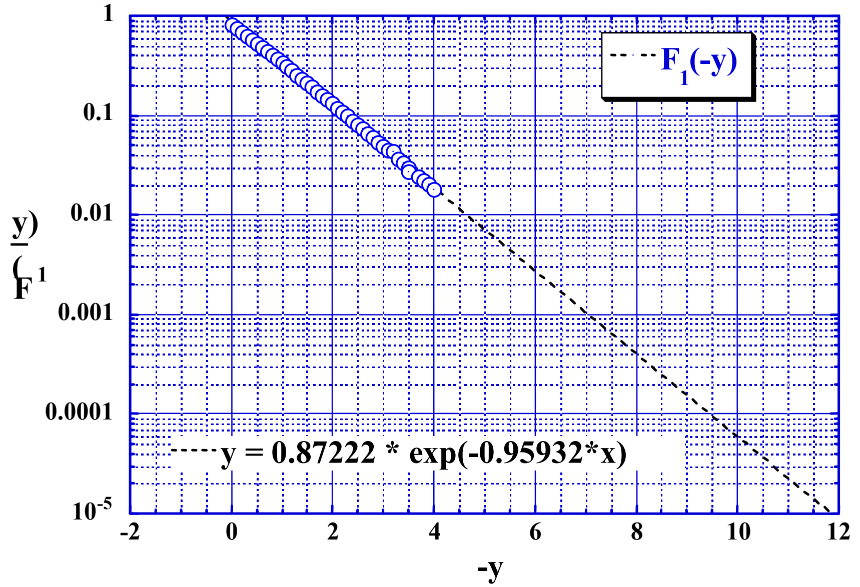
$L = 2.05E-8,\ RRR = 200,\ \rho_{295K} = 14.5E-8\ \Omega m,\ a = 7.52E-7$

$-y = \alpha\cdot\frac{T_c}{T},\ \alpha = 1.53,\ T_c = 9.25K,\ T \le 0.6\cdot T_c$

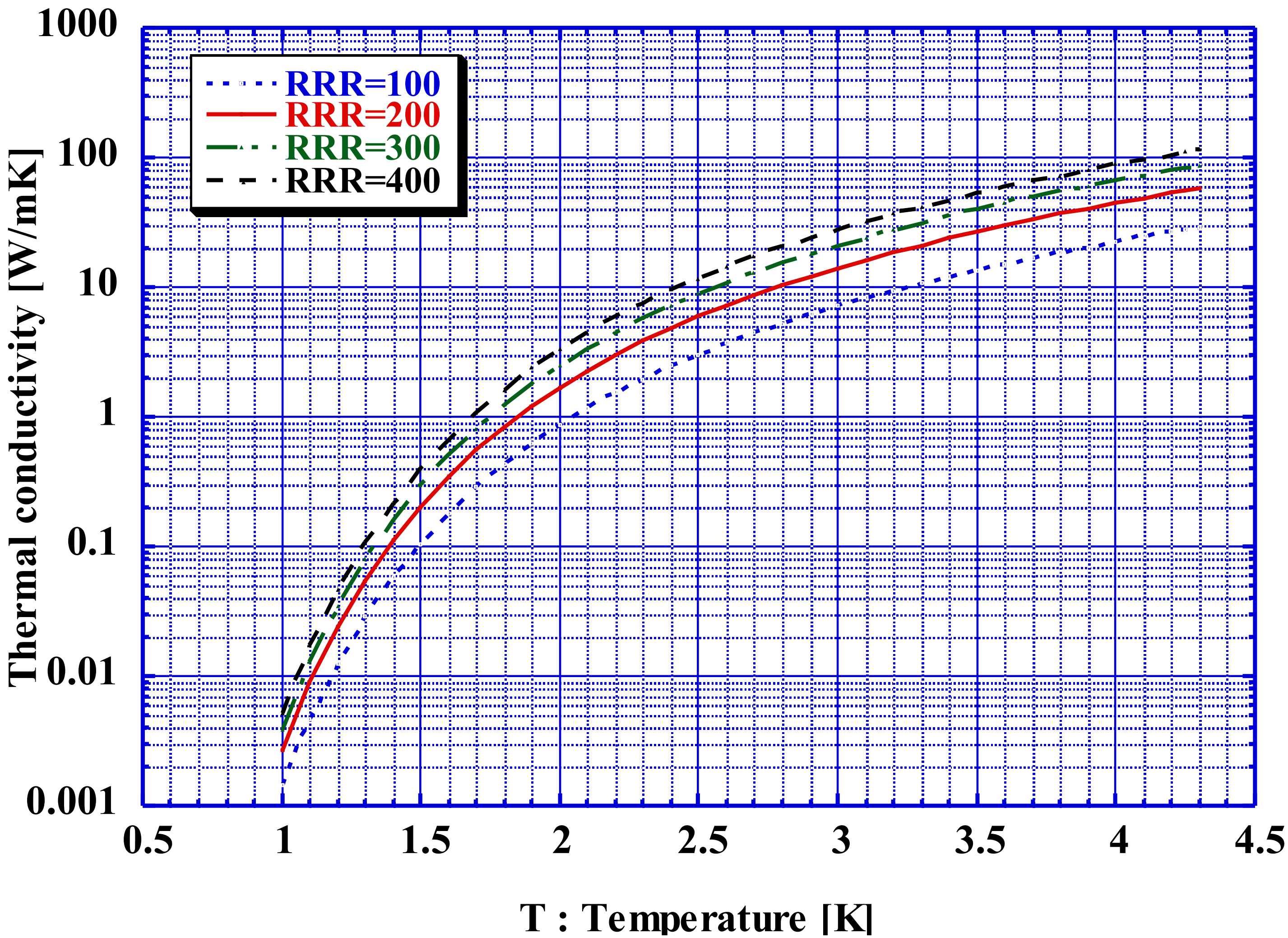
$D = 4.27E-3,\ B = 4.34E3,\ l = 50\mu m$

$$R(y) = \frac{\kappa_{es}}{\kappa_{en}} = \frac{2F_1(-y) + 2y\ln(1+e^{-y}) + \frac{y^2}{(1+e^y)}}{2F_1(0)},$$

$$F_n(-y) = \int_0^\infty \frac{z^n}{1+e^{z+y}}dz$$

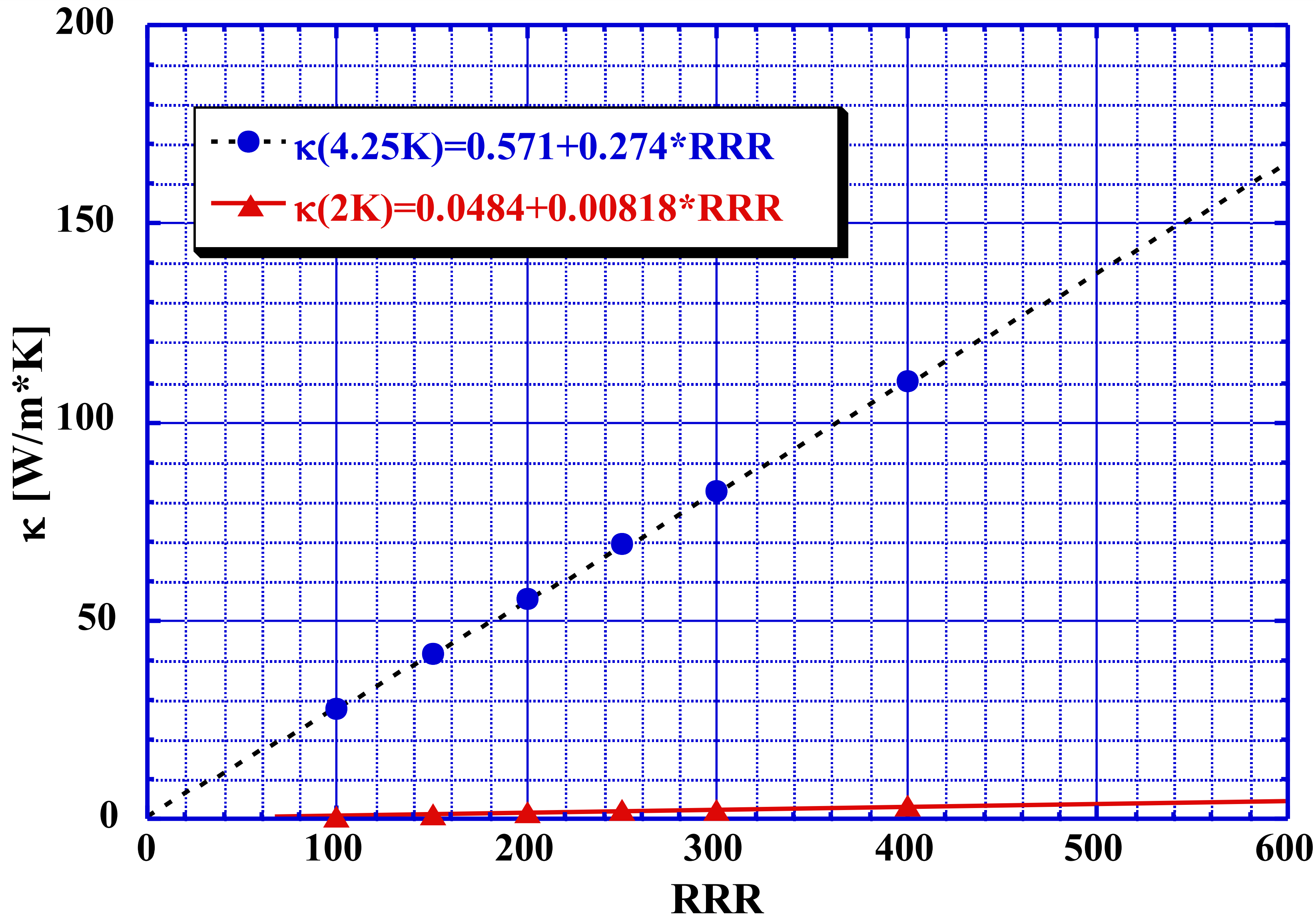


Calculated κ_{sc}(T) for Niobium



- Notice, the thermal conductivity of Nb at 2K is very small, which is a 1/6800 of pure copper material.

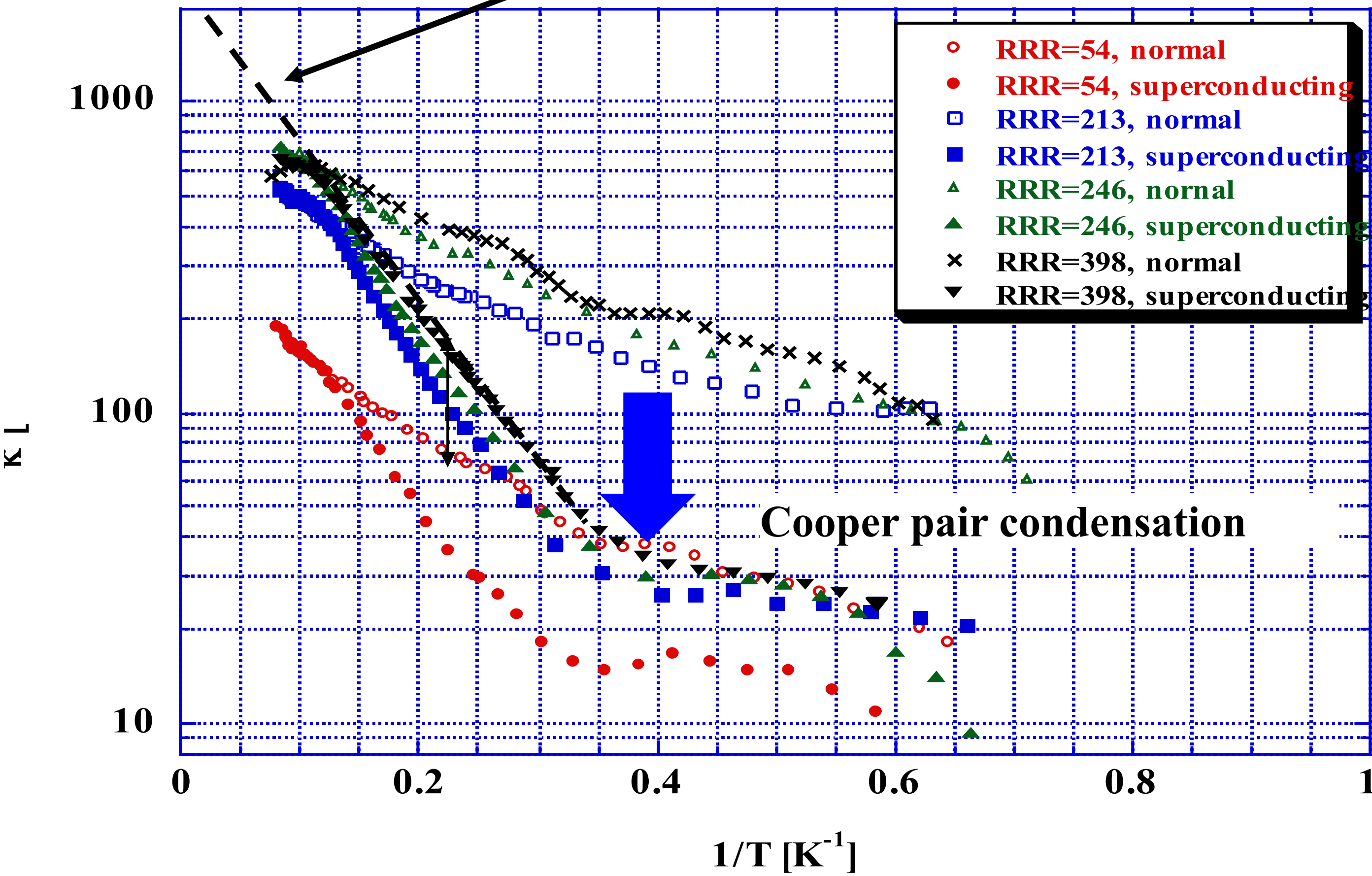
Linear relationship between κ_{sc} (2K, 4.25K) and RRR

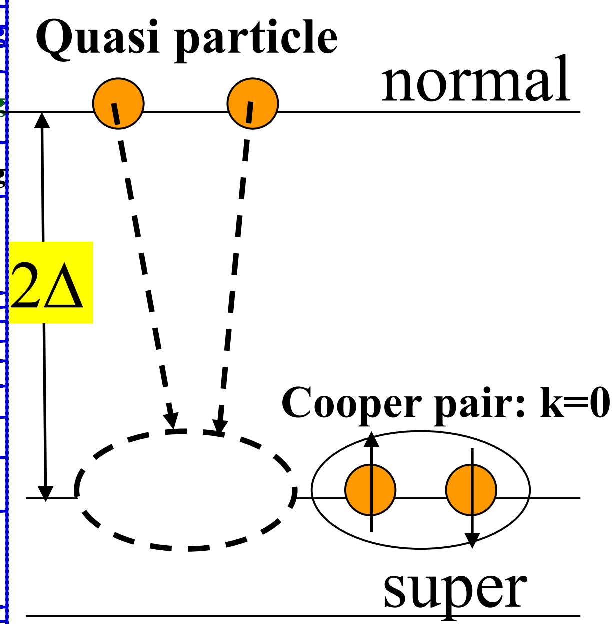


Poor Thermal Conductivity in SC State

Boltzmann statistics : energy Δ, Temp. T existing probability

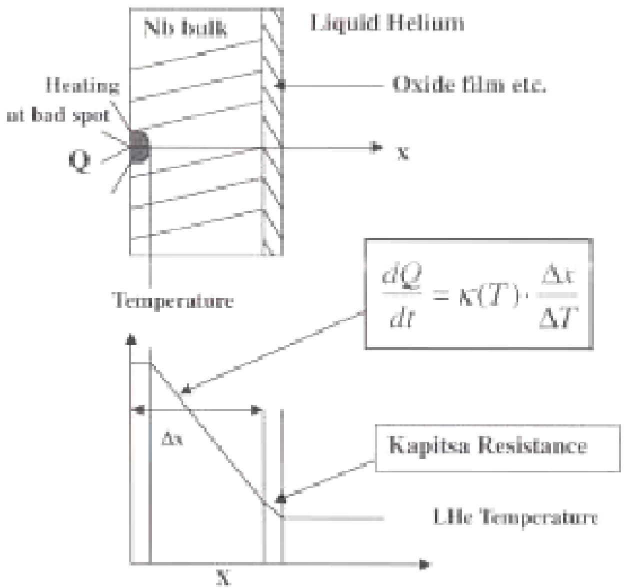
$$\exp\left(-\frac{\Delta}{k_B \cdot T}\right)$$





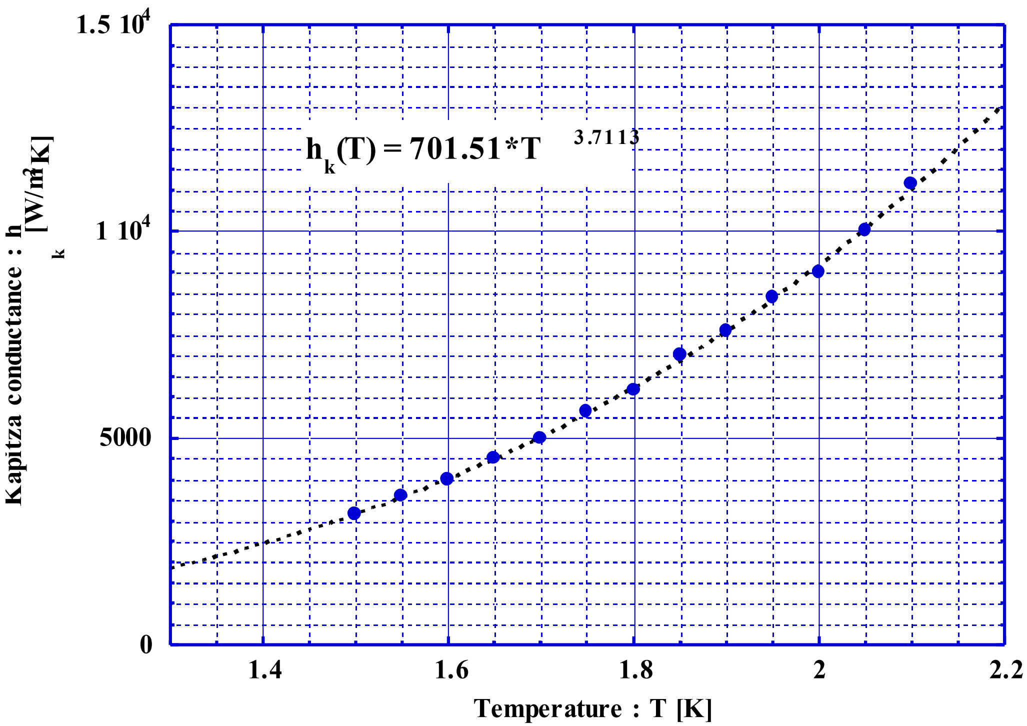
Kapitza Resistance

- Kapitza resistance disturbs the heat transfer at the interface of cavity wall outer surface/liquid Helium.
- Heavily oxidized cavity outer wall surface (for instance Ti oxide vaporized) needs a special care the high Kapitza resistance, but which has small impact with etched surface
- Kapitza heat transfer enhances with $T^{3.7}$ increased temperature.



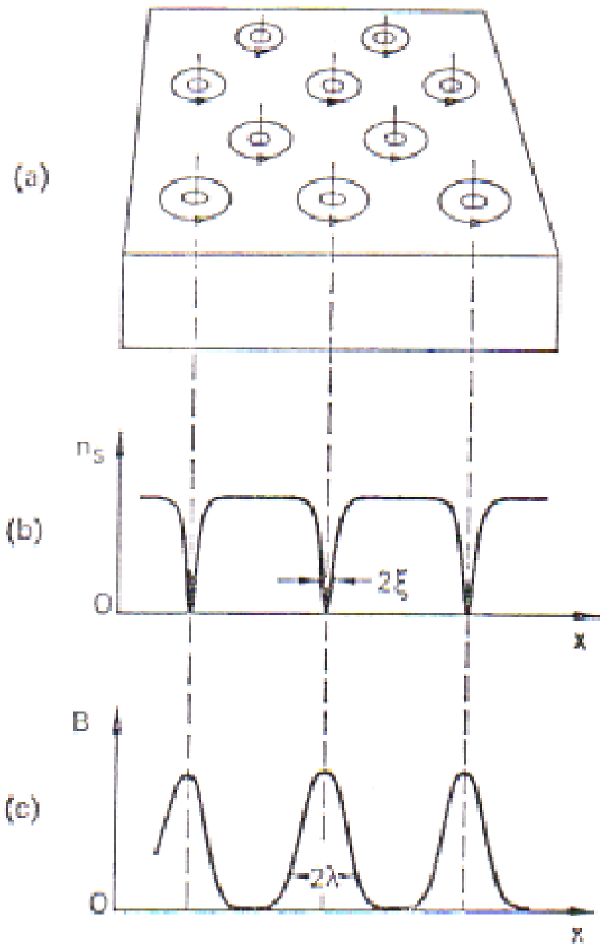
$$\Delta T = P\cdot\left\{\frac{d}{\kappa_{SC}(\tilde{T})} + \frac{1}{h_k(\tilde{T})}\right\}$$

$$h_k(T) = 705.51\cdot T^{3.7113}$$



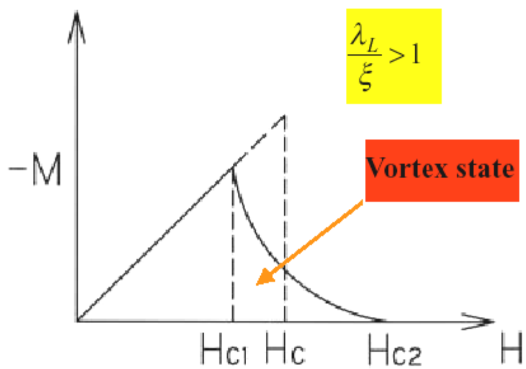
Vortex State

Observation of vortex using iron powders



Vortex state

ξ : Coherence length
size of Cooper pair



λ_L : London penetration depth

Depth of penetration of the magnetic field

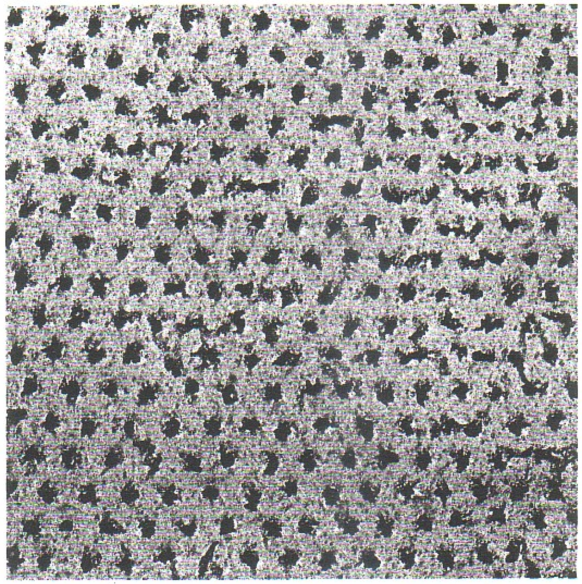


Figure 19 Triangular lattice of fluxoids through top surface of a superconducting cylinder. The points of exit of the flux lines are decorated with fine ferromagnetic particles. The electron microscope image is at a magnification of 8300, by U. Essmann and H. Träuble.

Trapped flux is quantized.

Observation of vortex (fluxiod) using Electron Phorography

Fluxiod: quantized flux

By A. Tonomura et al.

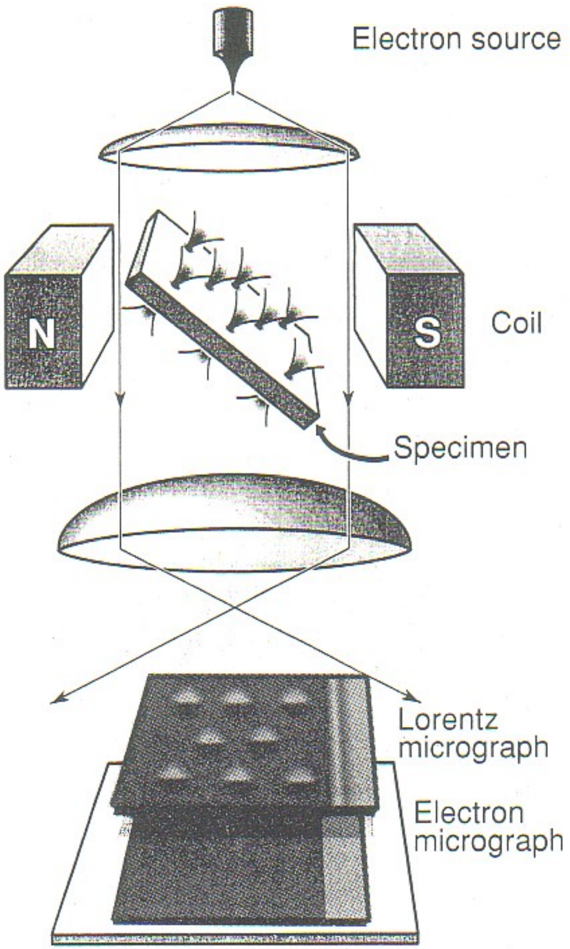
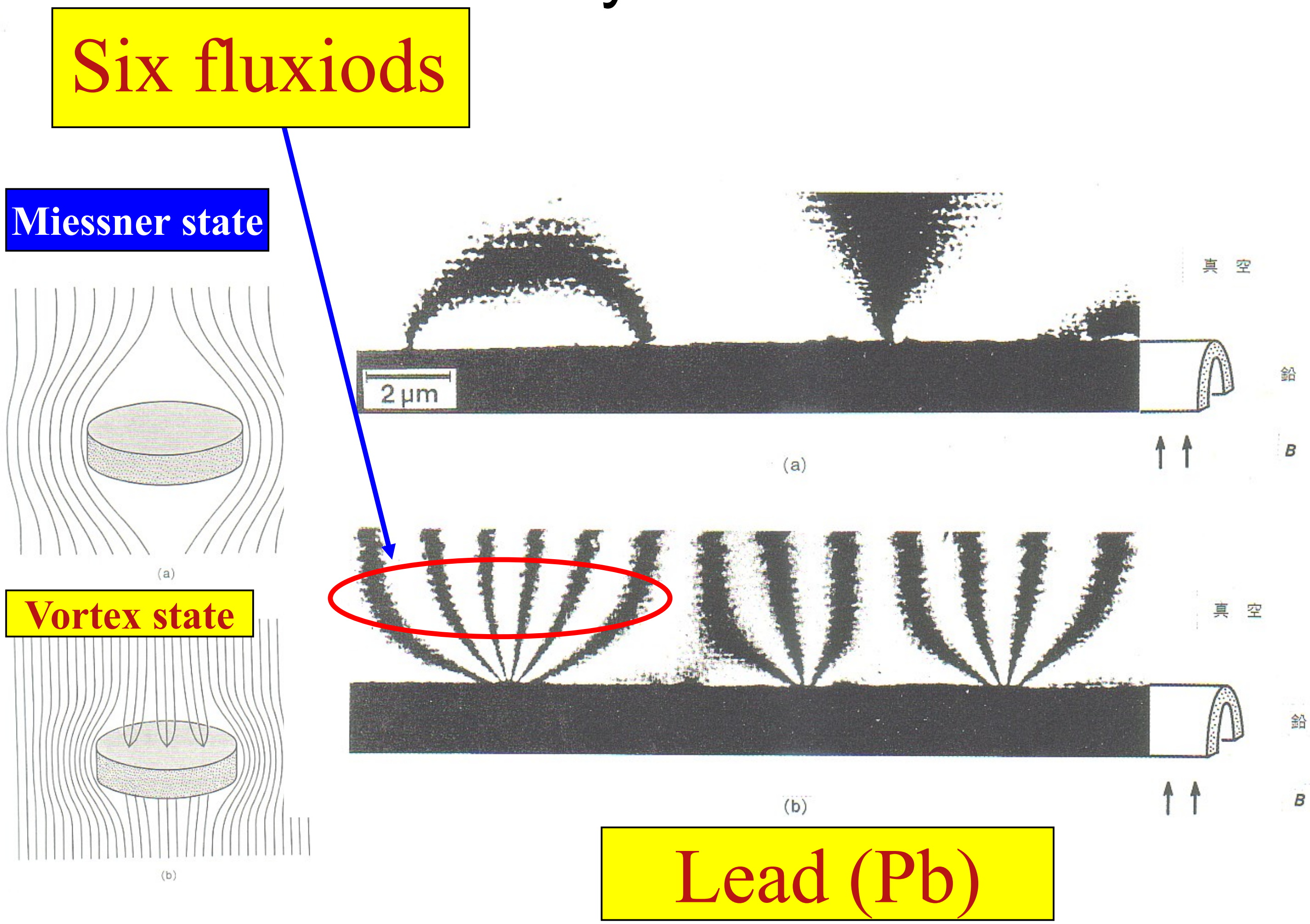


FIG. 1 Schematic diagram of vortex-lattice observation.



Vortex on Nb film

Vortex lattice on Nb film

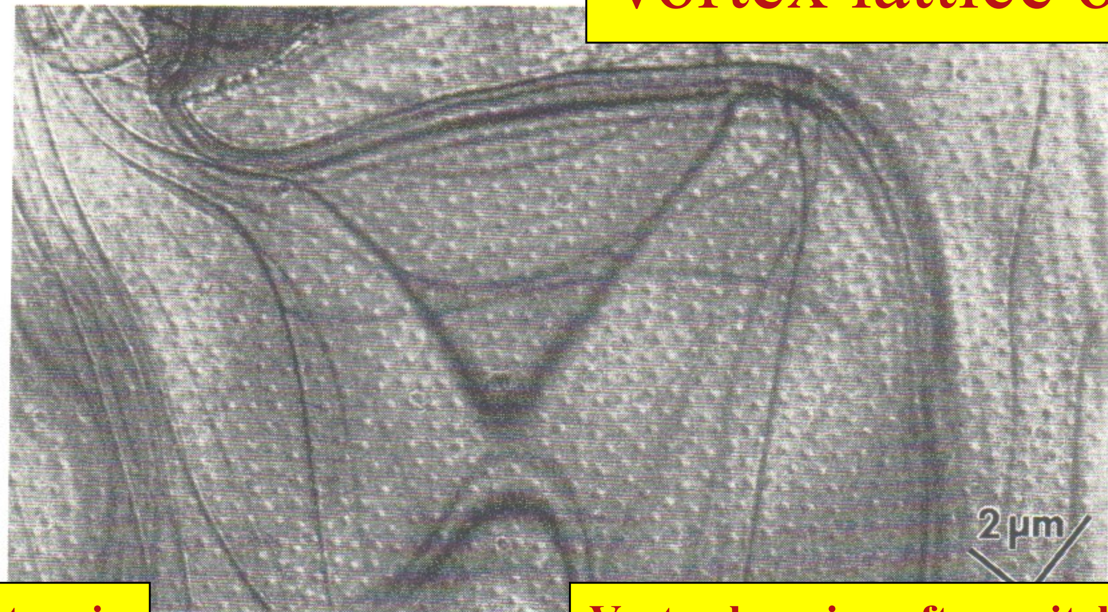


FIG. 2 Lorentz micrograph of a Nb thin film. A magnetic field of 100 G is applied to the film in the superconducting state at 4.5 K. Vortices are observed as small globules, each with black and white contrast. The magnification differs in the two directions by $\sqrt{2}$, because the film is tilted by 45°. The dark lines are bend contours.

T-dependence of vortex size

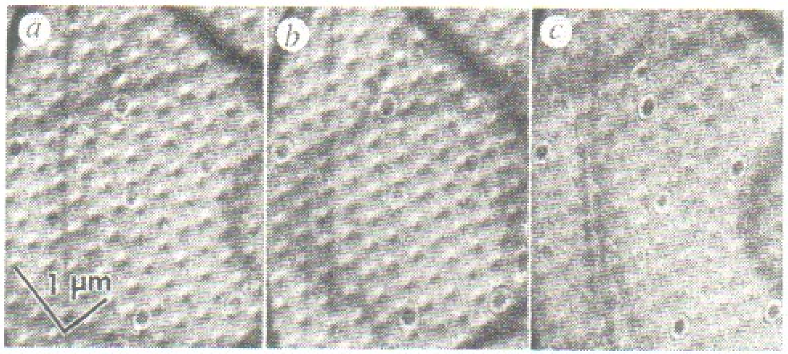


FIG. 3 Dependence of the vortex images on temperature. *a*, $T = 4.5$ K; *b*, $T = 6.0$ K; *c*, $T = 8.0$ K. The vortices become larger as the penetration depth increases with temperature. The vortex configuration, random at 4.5 K, turns into a more and more regular hexagonal lattice at higher T, which can be recognized more easily when the photographs are seen obliquely.

Vortex hopping after switched off

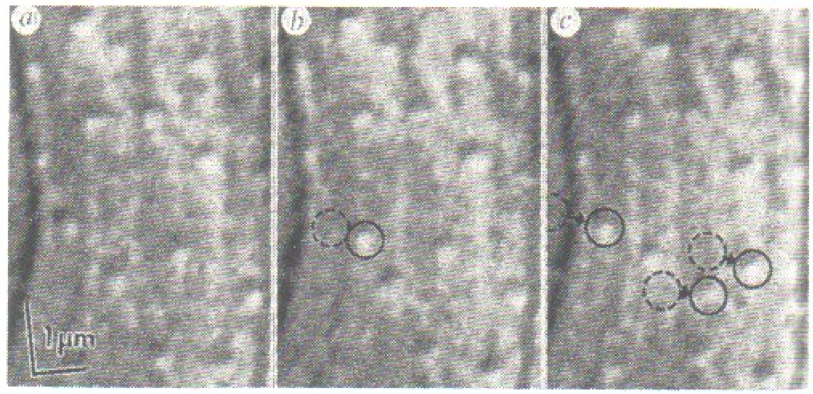
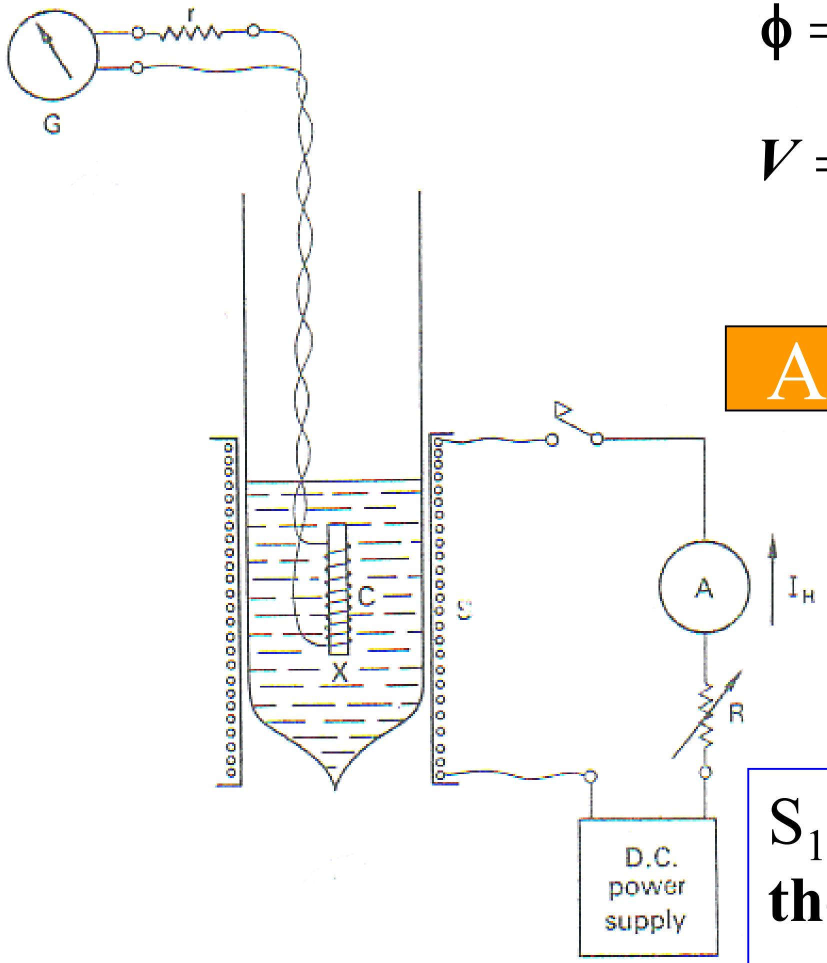


FIG. 4 Dynamics of the vortices as seen at times t after magnetic field has been switched off from 150 G. *a*, $t = t_0 + 0$ s; *b*, $t = t_0 + 0.1$ s; *c*, $t = t_0 + 1.4$ s ($t_0 = 170$ s). Dotted and solid circ… hopping, and the hopping directions are indicated by the arrows.

Measurement of the Critical Magnetic Field



$$\phi = \mu_0 B(t) \cdot [S_1 + S_s(t)], \; B(t) = B_0 t$$

$$V = -\frac{d}{dt}\phi = -\mu_0 B_0 S_1 - \mu_0 B_0 S_s - \mu_0 B_0 t \frac{d}{dt} S_s$$

Area of sample (flux penetrated)

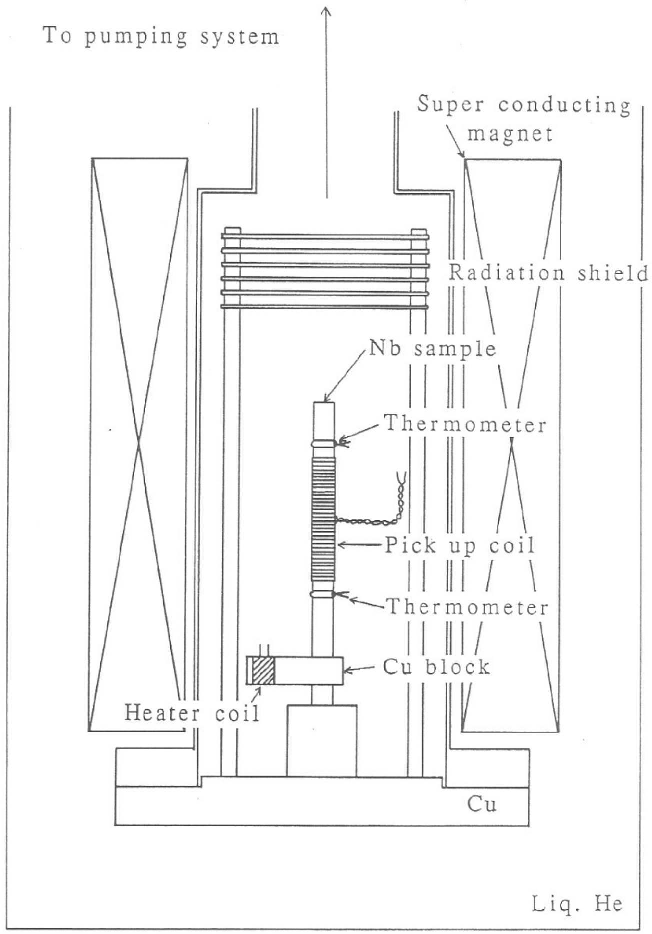
Cross-section

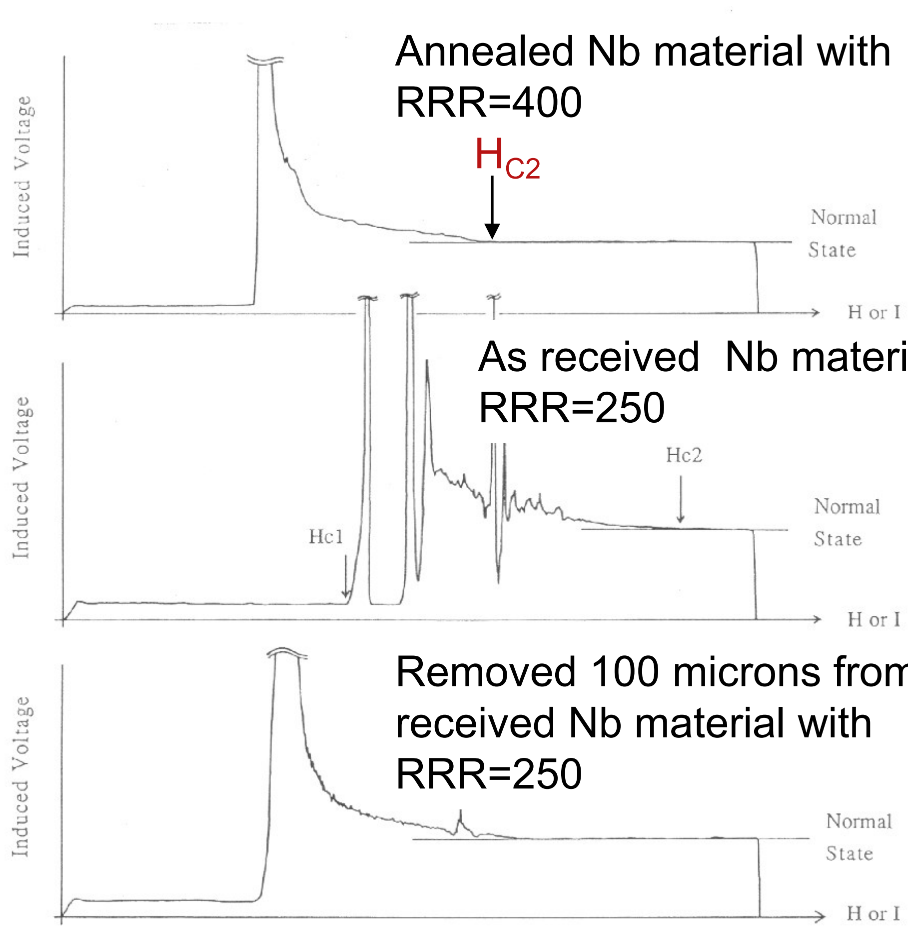
$S_S(t)$

S_1: a gap between the sample and coil

Coil

Example of signal by the simple measurement method





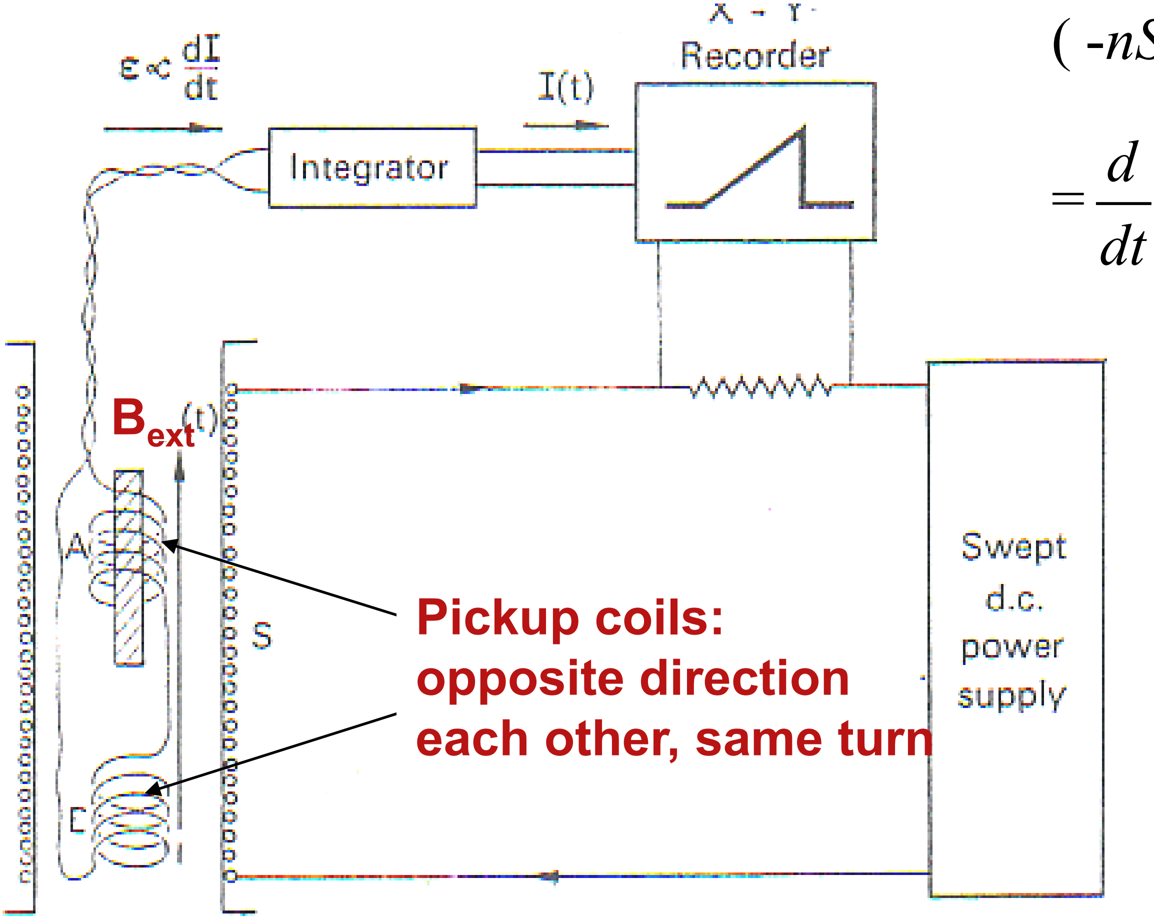
- Some offset signals produced by a gap between the sample and coil.
- Flux jumping phenomena with damaged or defect surface, or gain boundaries.
- This method can measure the depth of surface damage layer through magnetic property, ~ 30 microns.

More elegant measurement method of Hc

$$V = V_A + V_B = -\frac{d}{dt}\Phi_A + \frac{d}{dt}\Phi_B$$

$$(-nS_0\mu\frac{d}{dt}B_{ext} + \frac{d}{dt}M) + nS_0\mu\frac{d}{dt}B_{ext}$$

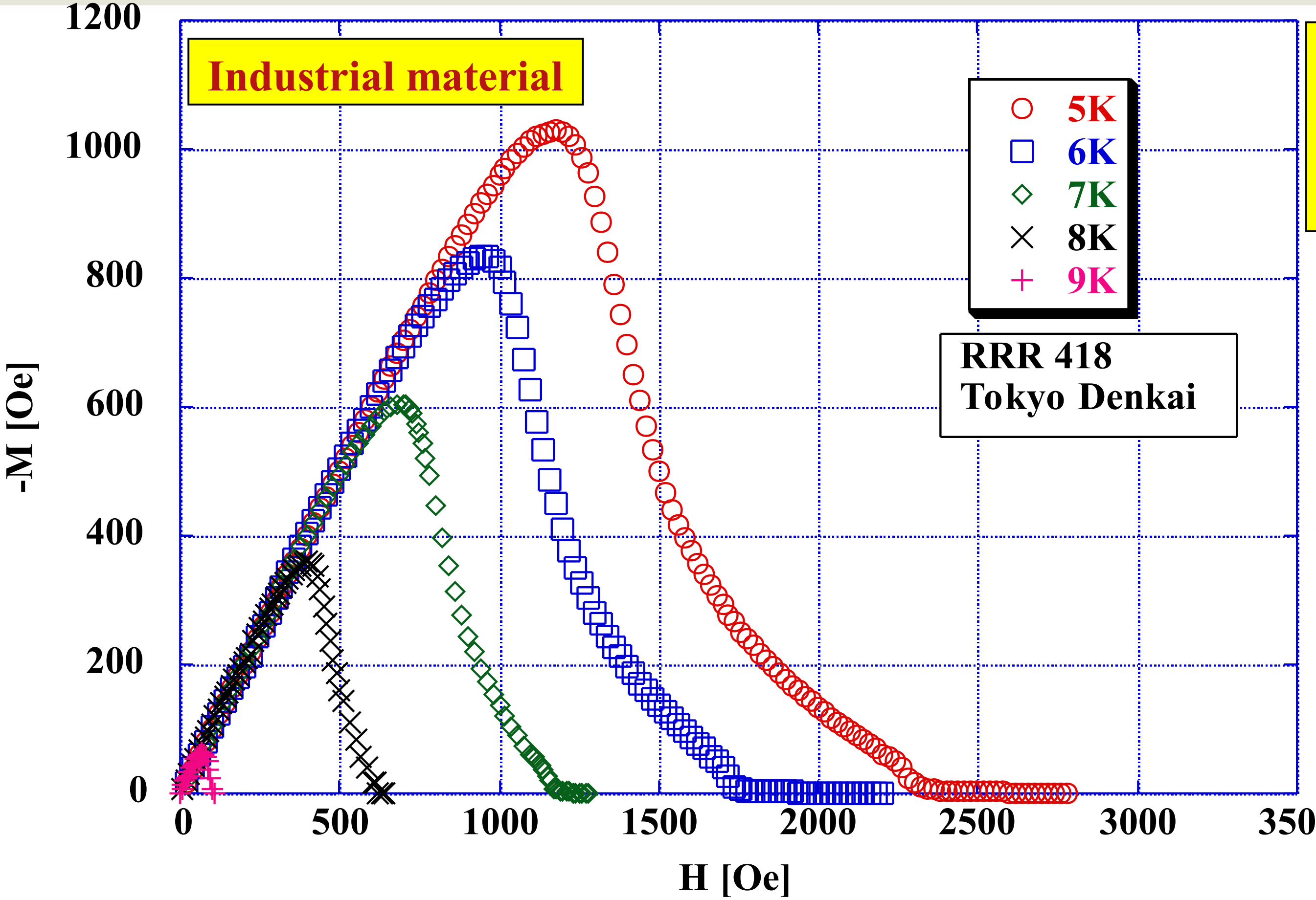
$$= \frac{d}{dt}M$$



$$M = \int_0^t V dt$$

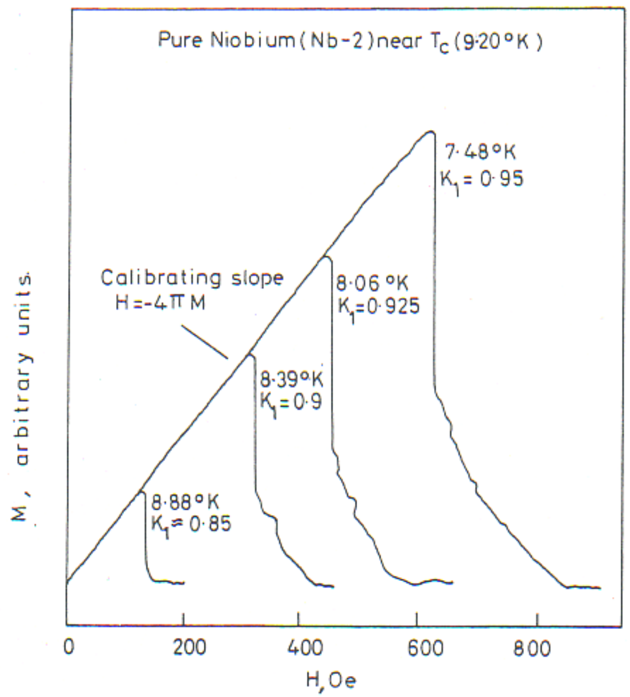
Can subtract the offset signal

Signal by the elegant measurement method



How different between industrial products and Lab product (very high purified material)

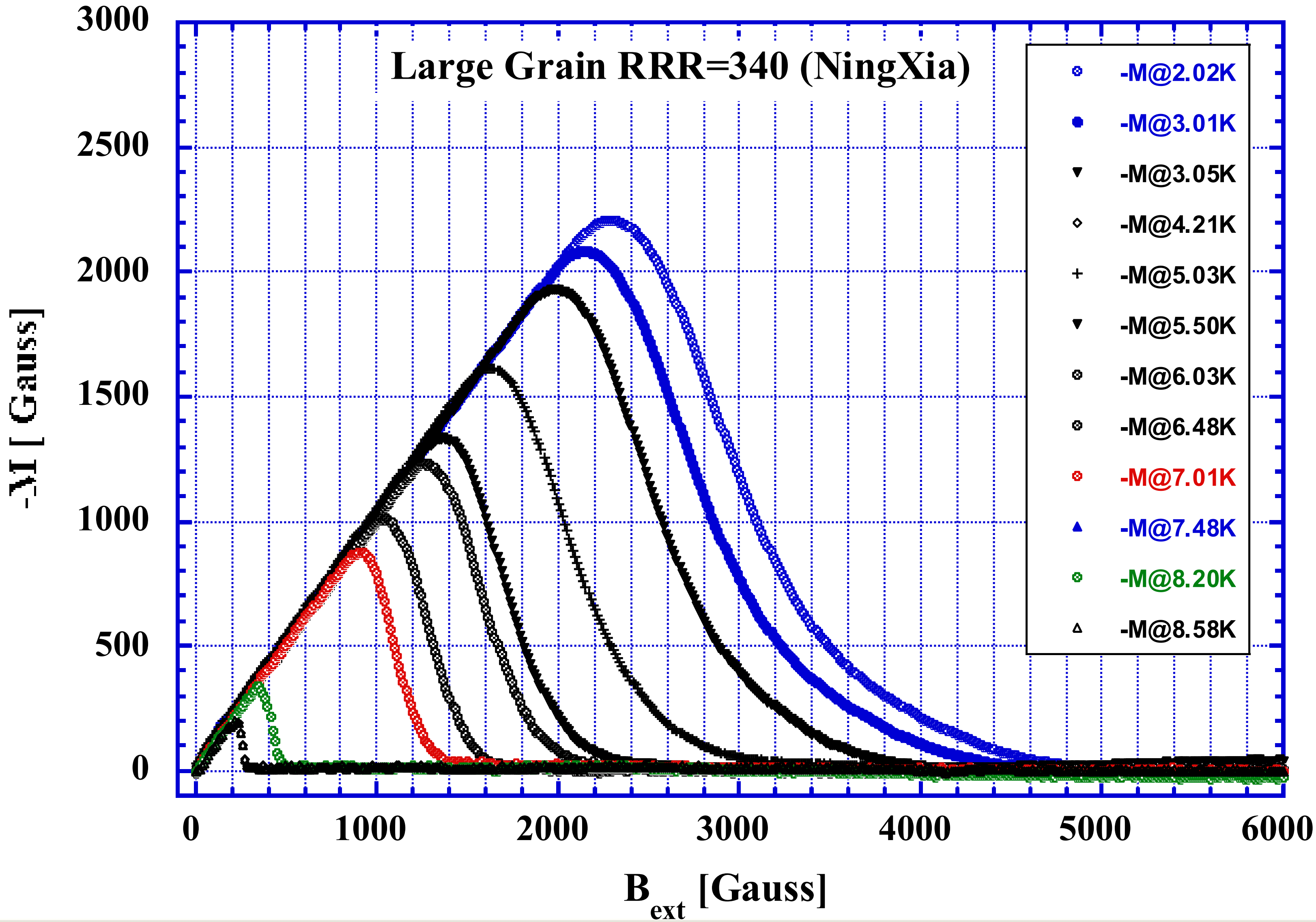
Lab material RRR~2000



$$F_n - F_s = -\int_0^{H_{c2}} M dH = \frac{1}{2}\mu H_c^2$$

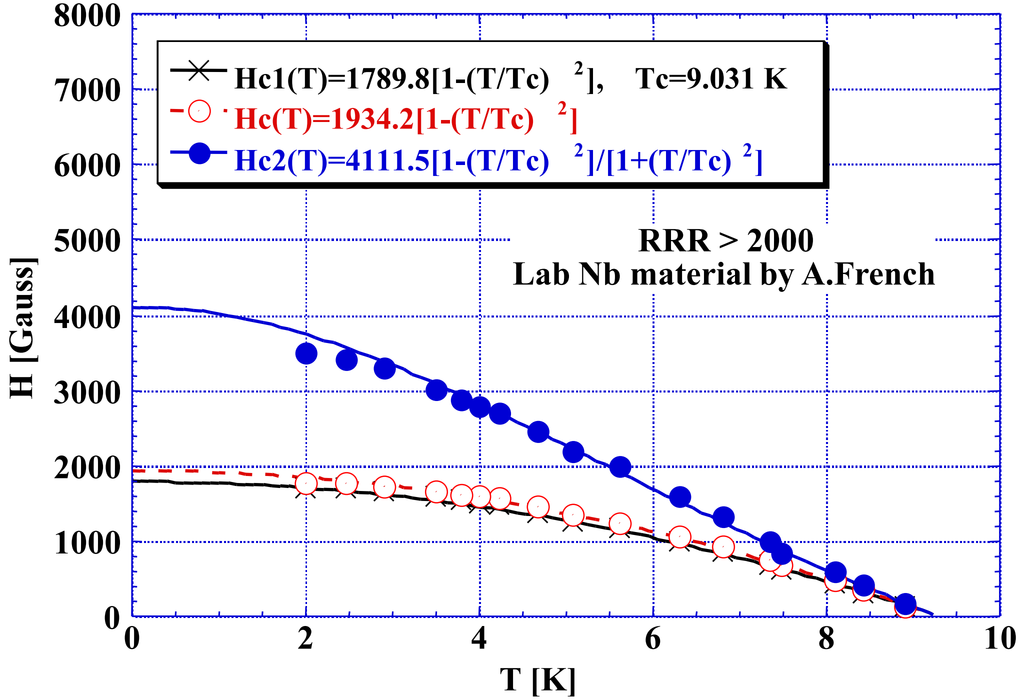
- High purity Nb is a type-II superconductor but very close to type-I.

Example of demagnetization curve on Niobium
(Ningxia, Large Grain RRR=340)

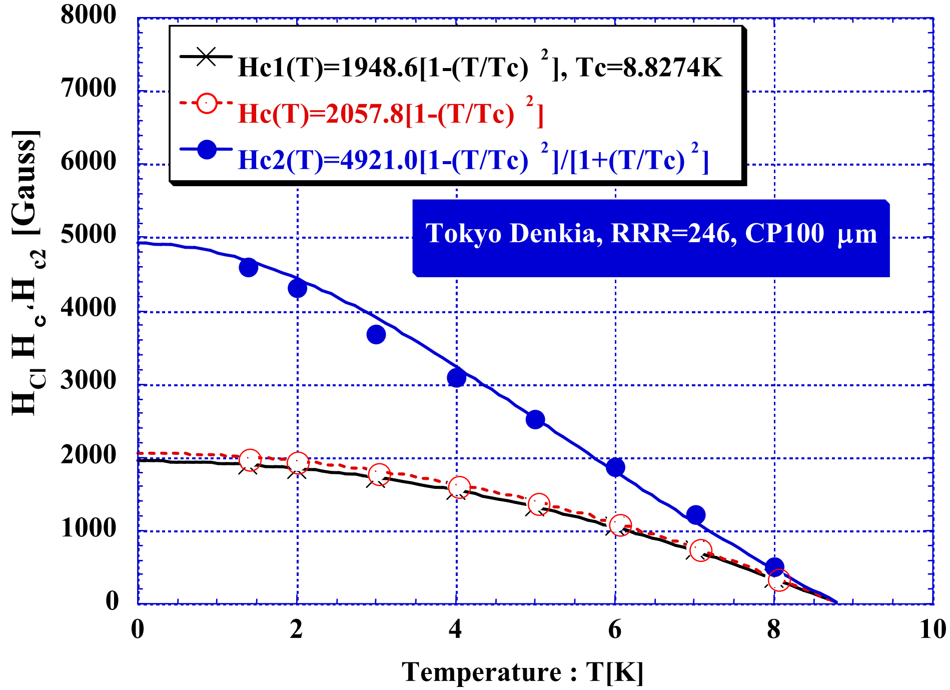


Measurement results with H_{C1}, H_C, H_{C2}

Lab Nb material



Industrial produced Nb material



$$H_c(T) = H_c(0)\cdot\left[1-\left(\frac{T}{T_c}\right)^2\right],\quad F_n - F_s = -\int_0^{H_{c2}} MdH = \frac{1}{2}\mu H_c^2$$

5.4 Abrikosov's Theory: Theory for Type-II Superconductors

$$H_c = \frac{\kappa}{\lambda^2}\frac{\hbar c}{\sqrt{2}e^*} = \frac{\kappa}{\lambda^2}\frac{(hc/2e)}{2\pi\cdot\sqrt{2}} = \frac{\phi_0}{2\pi\cdot\sqrt{2}\lambda\xi}$$

$$H_{c2} = \sqrt{2}\frac{\lambda}{\xi}\frac{\phi_0}{2\pi\cdot\sqrt{2}\lambda\xi} = \frac{\phi_0}{2\pi\xi^2}$$

$$H_{c1} = \frac{\phi_o}{4\pi\lambda^2}\ln\left(\frac{\lambda}{\xi} + 0.08\right)$$

$$\phi_0 = hc/2e = 2.0678\times 10^{-7}\,Gauss\cdot cm^2$$

$$= 2.0678\times 10^{-15}\ T\cdot m^2$$

What limits the RF Critical Field ?

Abrikosov theory

$$\xi = \sqrt{\frac{\phi_o}{2\pi \cdot H_{c2}}} \;,\; \lambda = \sqrt{\frac{\phi_o \cdot H_{c2}}{4\pi \cdot H_c^2}}$$

Established by measurements:

$$\lambda(T) = \frac{\lambda(0)}{\sqrt{1 - \left(\frac{T}{T_c}\right)^4}} \;,\; H_c(T) = H_c(0) \cdot \left[1 - \left(\frac{T}{T_c}\right)^2\right]$$

$$H_{c2}(T) = \frac{4\pi\lambda(T)^2}{\phi_o} \cdot H_c(T)^2 = \frac{4\pi\lambda(0)^2 \cdot H_c(0)^2}{\phi_o} \cdot \frac{\left[1-(T/T_c)^2\right]^2}{1-(T/T_c)^4}$$

$$= H_{c2}(0) \cdot \frac{1-(T/T_c)^2}{1+(T/T_c)^2}$$

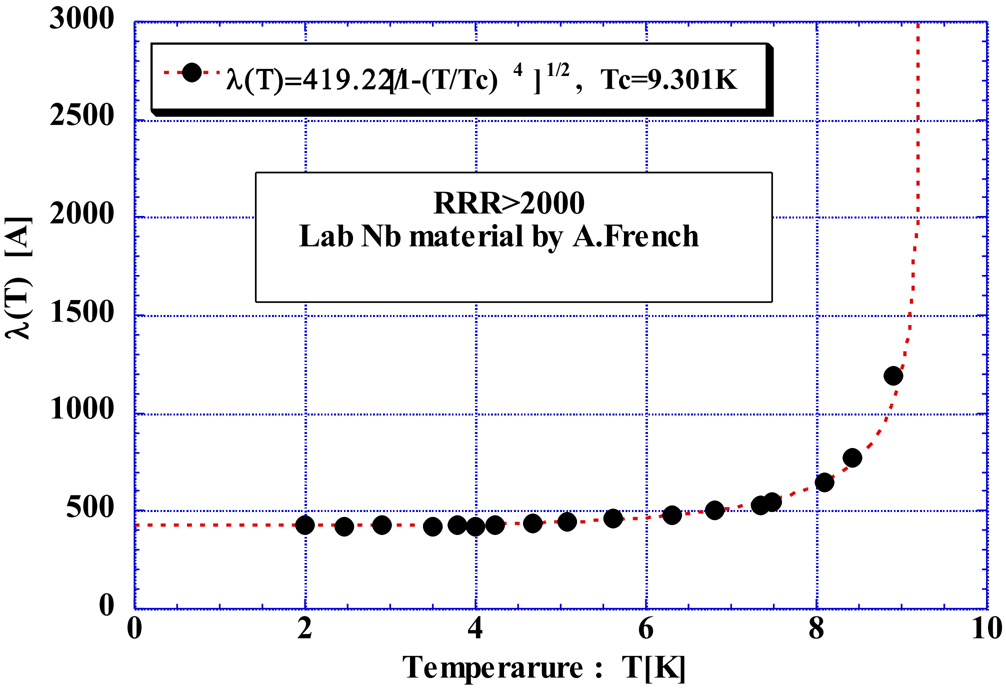
$$\xi(T) = \sqrt{\frac{\phi_o}{2\pi \cdot H_{c2}(0)}} \cdot \sqrt{\frac{1+(T/T_c)^2}{1-(T/T_c)^2}} = \xi(0) \cdot \sqrt{\frac{1+(T/T_c)^2}{1-(T/T_c)^2}}$$

$$\frac{\lambda(T)}{\xi(T)} \equiv \kappa(T) = \frac{1}{\sqrt{2}} \cdot \frac{H_{c2}(T)}{H_c(T)} = \frac{H_{c2}(0)}{\sqrt{2} \cdot H_c(0)} \cdot \frac{1}{1+(T/T_c)^2} = \frac{\kappa(0)}{1+(T/T_c)^2}$$

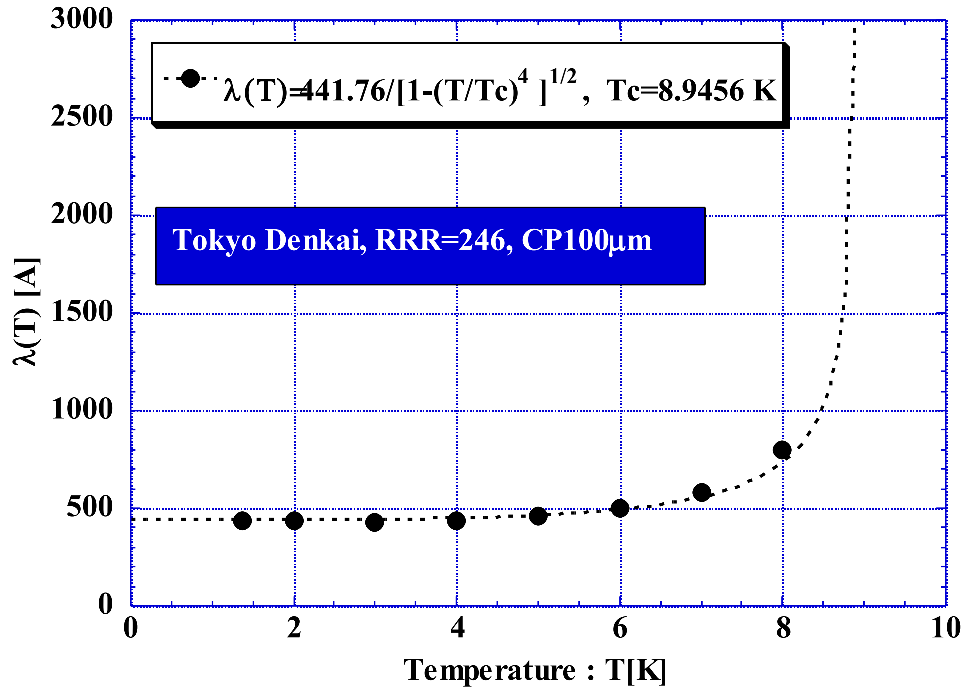
5.4 SC magnetic properties for very high purity lab niobium specimens and industrial procured niobium material

RF penetration depth λ

Lab material, RRR>2000



Industrial produced material

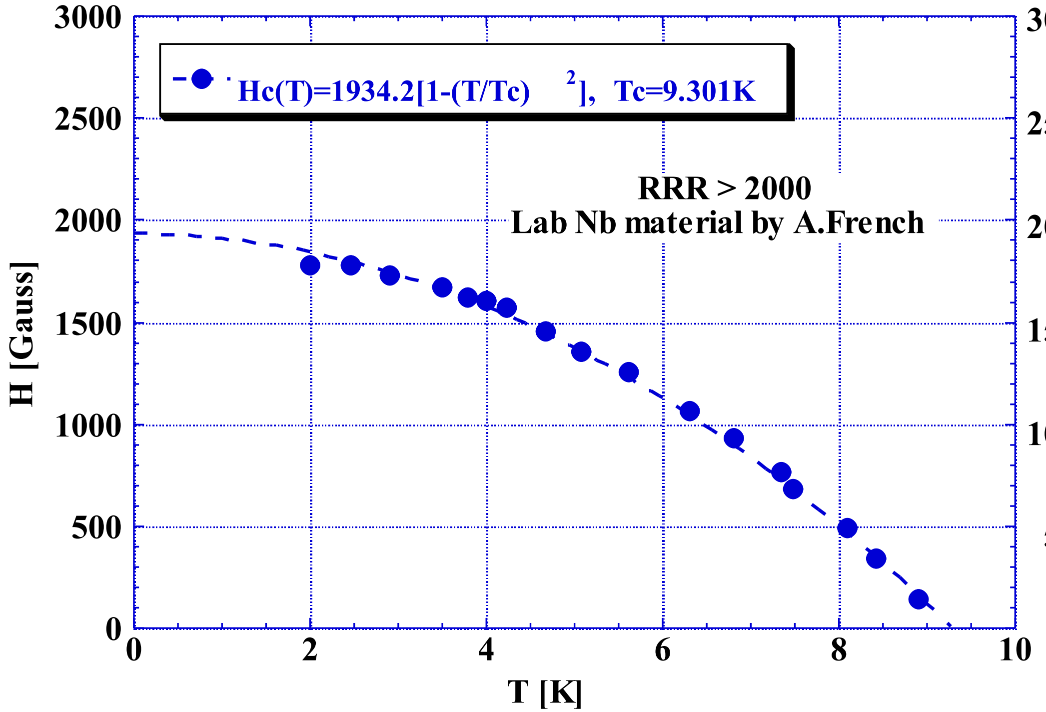


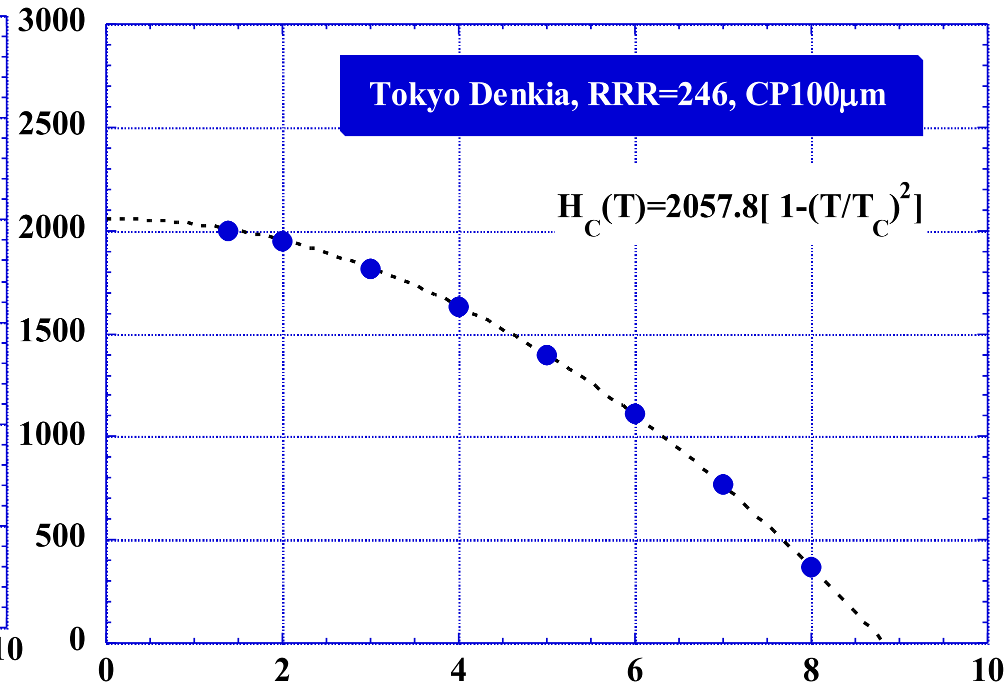
- Both lab and industrial materials behave as theory expected.

H_C Fitting

Lab material, RRR>2000

Industrial produced materail



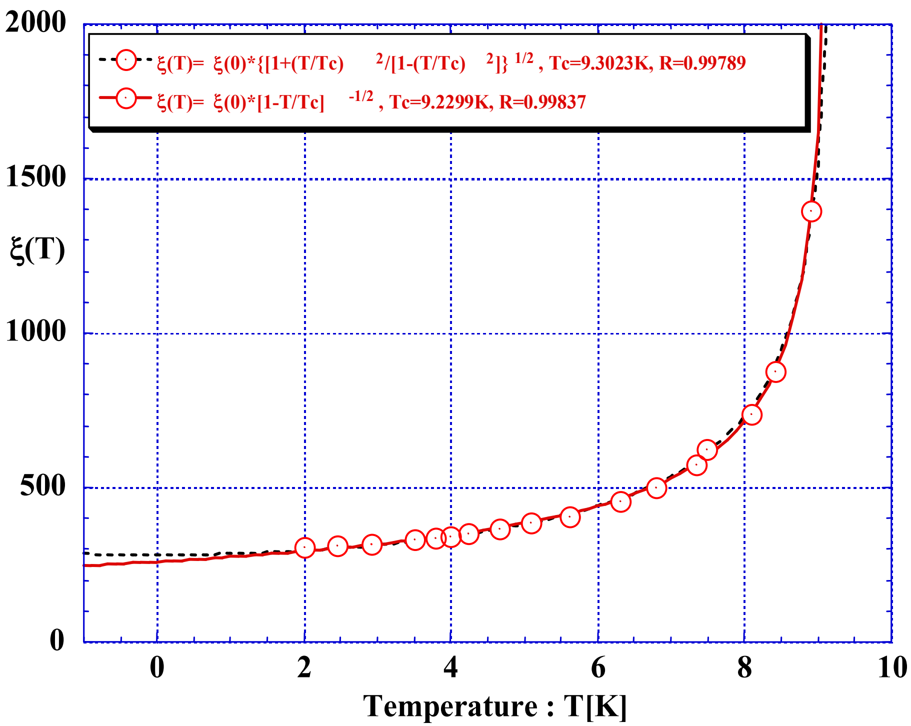


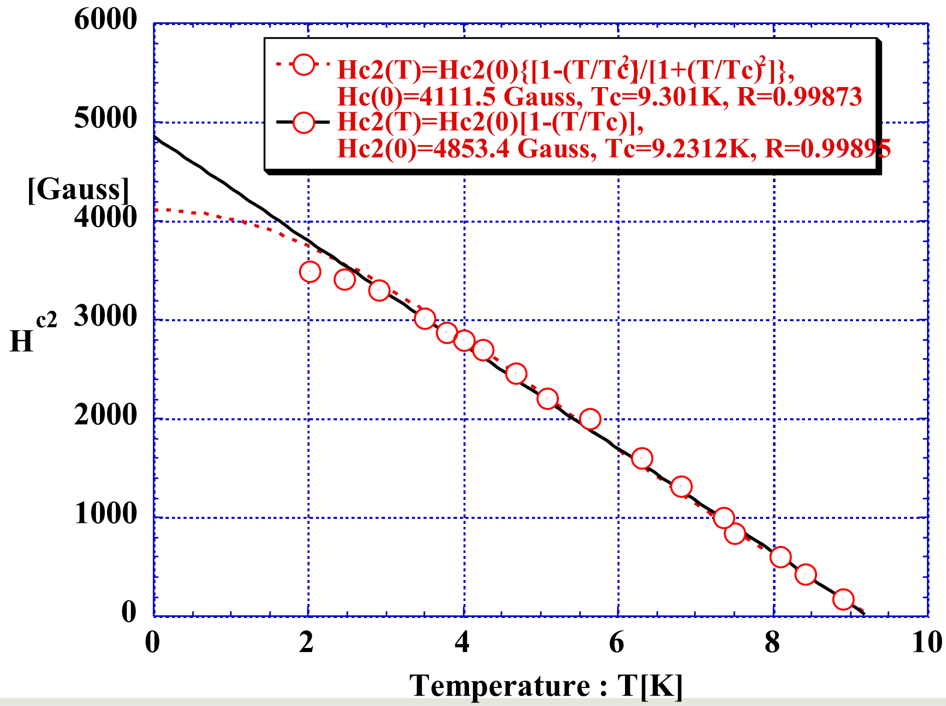
- Both lab and industrial materials behave as theory expected.

H_{C2}(T), ξ(T) fit for Lab material, RRR>2000

- The formulas H_{C2}(T) and ξ(T) given by Abrikosov's theory can fit the experimental results very well.

$$H_{C2}(T) = H_{C2}(0)\cdot\frac{1-\left(\frac{T}{T_C}\right)^2}{1+\left(\frac{T}{T_C}\right)^2} \qquad \xi(T) = \xi(0)\cdot\sqrt{\frac{1+\left(\frac{T}{T_C}\right)^2}{1-\left(\frac{T}{T_C}\right)^2}}$$

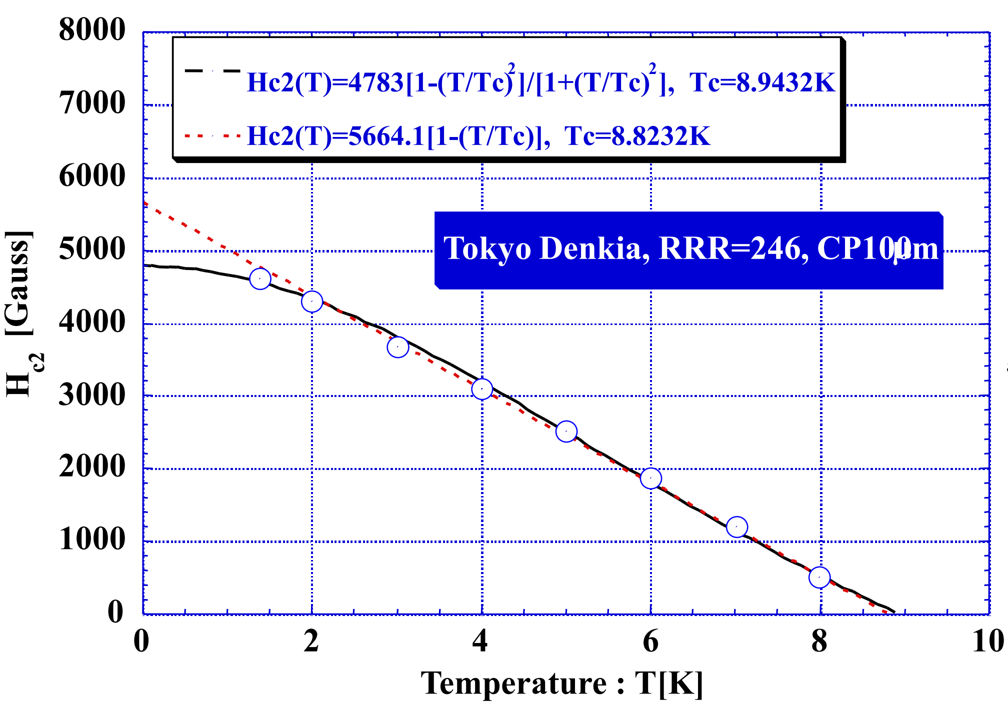


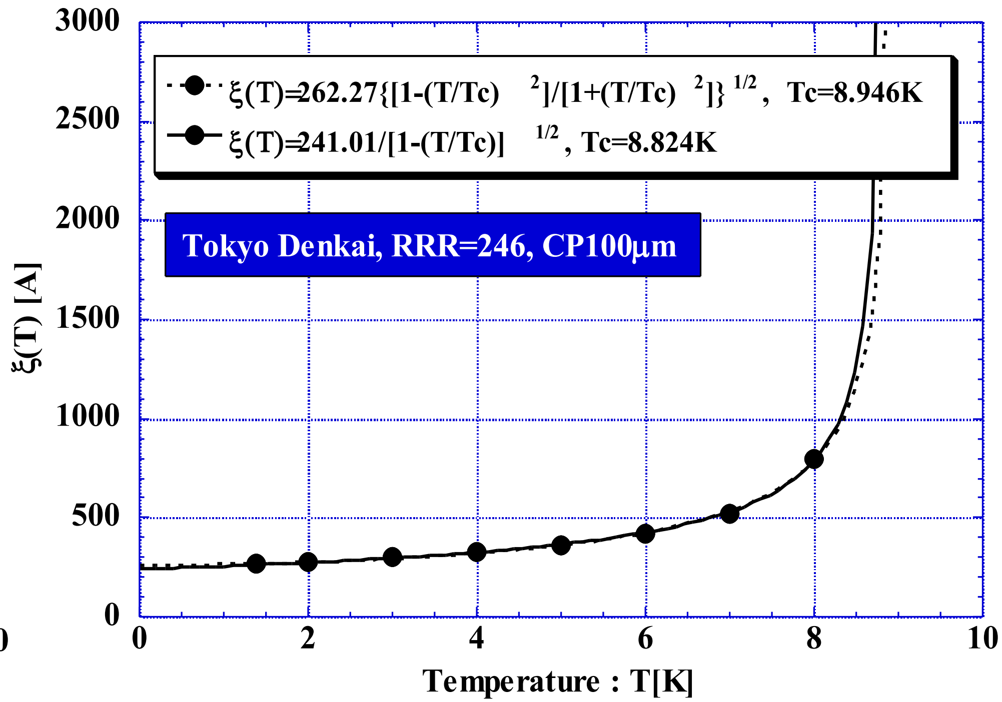


H_{C2} and ξ fitting for industrial material

$$H_{C2}(T) = H_{C2}(0)\cdot\frac{1-\left(\frac{T}{T_C}\right)^2}{1+\left(\frac{T}{T_C}\right)^2} \qquad \xi(T) = \xi(0)\cdot\sqrt{\frac{1+\left(\frac{T}{T_C}\right)^2}{1-\left(\frac{T}{T_C}\right)^2}}$$

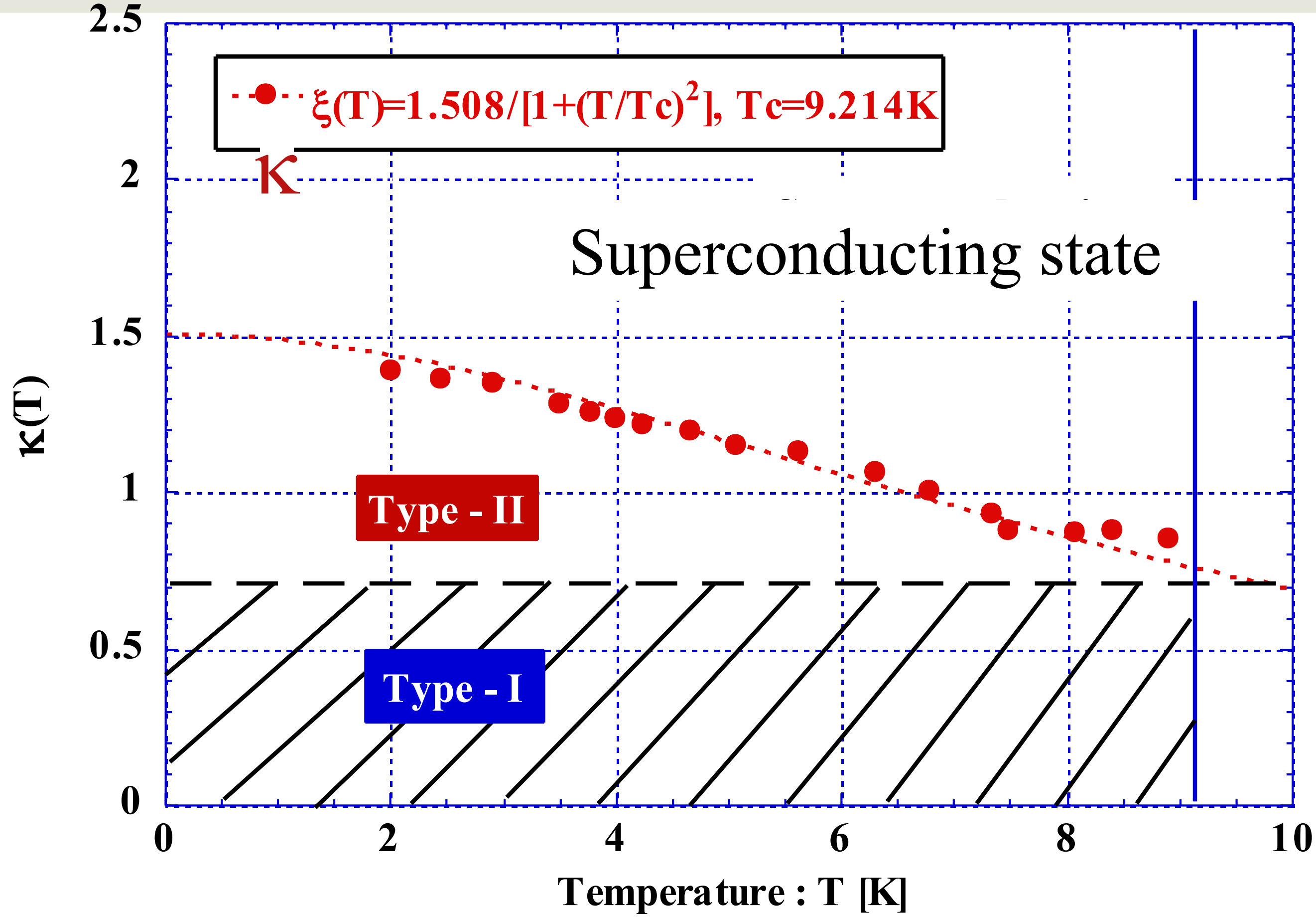
Above formulas $H_{C2}(T)$ and $\xi(T)$ can fit the industrial material too.





T-dependence of κ with Lab material

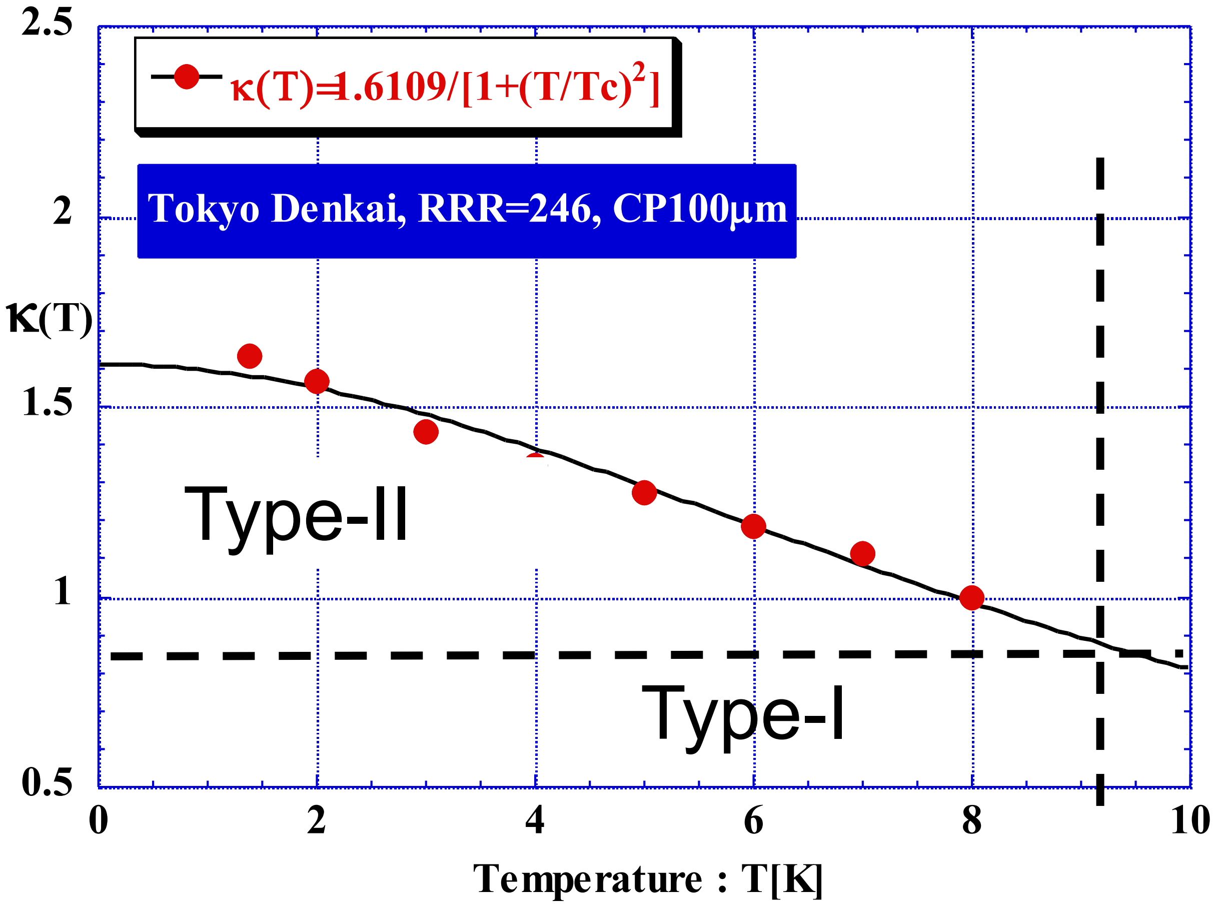
Nb
Lab material
RRR>2000



- κ is less than $\frac{1}{\sqrt{2}}$ at over all T < T_C (9.25K) with very high purity lab produced Nb material, which means type-II superconductor.

T dependence of κ with industrial material

RRR=246



- κ is less than $\frac{1}{\sqrt{2}}$ at over all $T < T_C$ (9.25K) with high purity industrial Nb material, which means type-II superconductor.

5.4 Why surface resistance in SRF application

Let's start normal conductor case.

Maxwell equations

$$\nabla \cdot \vec{B} = 0\,,\ \nabla \times \vec{E} + \mu \frac{\partial \vec{H}}{\partial t} = 0$$

$$\nabla \cdot \vec{D} = 0\,,\ \nabla \times \vec{H} - \varepsilon \frac{\partial \vec{E}}{\partial t} - \sigma \vec{E} = 0$$

$$\vec{J} = \sigma \vec{E}\ \text{(Ohm's law)}$$

$\vec{E}, \vec{H}$ are functions depending on only z, and t.

Decomposed into longitudinal (l) ant transvers components (t):

$$\vec{E}\,(z,t) = \vec{E_l}(z,t) + \vec{E_t}(z,t),\ \vec{H}\,(z,t) = \vec{H_l}(z,t) + \vec{H_t}(z,t)$$

Longitudinal components:

From Maxwell eq.

$$\frac{\partial H_\ell}{\partial z} = 0\ (\because \nabla \cdot \vec{B} = 0) \Longrightarrow \vec{H_l}(z,t) \text{ is not 0 for only static field.}$$

$$\left\{\frac{\partial}{\partial t} + \frac{\sigma}{\varepsilon}\right\} \vec{E_l} = 0 \Longrightarrow E_l = E_l(0) \cdot e^{-\sigma t/\varepsilon}\text{, which damps fast for good conductors}$$

Transverse Component and Skin Depth

Transverse components:

Supposed a plane wave $\overrightarrow{E_t} = \overrightarrow{E_0} \cdot \exp(i\, \vec{k} \cdot \vec{x} - i\, \omega t)$

From the Maxwell first rotation eq. $\overrightarrow{H_t} = \frac{1}{\mu_0 \omega} (\vec{k} \times \overrightarrow{E_t})$

From the Maxwell second rotation eq. $i(\vec{k} \times \overrightarrow{E_t}) + i\varepsilon_0 \omega \overrightarrow{E_t} - \sigma \overrightarrow{E_t} = 0$

Removed $\overrightarrow{E_t}$ or $\overrightarrow{H_t}$

$$[k^2 - (\mu_0 \varepsilon_0 \omega^2 + i\mu_0 \varepsilon_0 \sigma)] \cdot \begin{Bmatrix} \overrightarrow{E_t} \\ \overrightarrow{H_t} \end{Bmatrix} = 0$$

$$k^2 = \mu_0 \varepsilon_0 \omega^2 \left(1 + i \frac{\sigma}{\omega \varepsilon_0}\right)$$

$$k = \alpha + i\beta$$

$$\begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \sqrt{\mu_0 \varepsilon_0} \omega \cdot \left[\frac{\sqrt{1 + \left(\frac{\sigma}{\omega \varepsilon_0}\right)^2} \pm 1}{2} \right]^{1/2}$$

For good conductors

$$\frac{\sigma}{\omega \varepsilon_0} >> 1, \quad k \sim \left(1 + i \sqrt{\frac{\mu_0 \sigma \omega}{2}}\right)$$

Skin depth:

$$\delta = \frac{1}{\beta} = \sqrt{\frac{2}{\sigma \omega \mu_0}}$$

Surface Impedance

$$k = (1+i)\sqrt{\frac{\mu\sigma\omega}{2}}$$

$$\overrightarrow{H_t} = \frac{1}{\omega\mu_0}\cdot\vec{k} \times \overrightarrow{E_t}$$

Impedance Z: $Z \equiv \frac{E_t}{H_t} = R_S + iX_S = \frac{\omega\mu_0}{k}$, $k = (1+i)\sqrt{\frac{\sigma\omega\mu_0}{2}}$

$\vec{k}$ and $\overrightarrow{E_t}$ are orthogonal.

Surface impedance:

$$Z = (1-i)\sqrt{\frac{\omega\mu_0}{2\sigma}}$$

Surface resistance for good conductors

$$R_S = \sqrt{\frac{\omega\mu_0}{2\sigma}} = \frac{1}{\sigma\delta}$$

Frequency Dependence of d and Rs with Copper Cavity

Frequency	δ[μm]	Rs [mΩ]	Comment
508MHz	2.90	5.78	TRISTAN
1300MHz	1.81	9.24	ILC
2856MHz	1.22	13.70	Conventional LINAC
11.4GHz	0.61	27.36	GLC

Calculated for $\sigma = 59.6 \times 10^6$ S/m (Cu)

Note:

- The higher frequency penetrates the shallower skin depth.
- The higher frequency makes the higher surface resistance.

Surface Resistance for Superconductors

- The correct derivation of the surface resistance is done by BCS theory for superconductivity case. However, it is very complicated.
- Two fluid model is simple and includes the fundamental physics. Here, why SRF surface resistance appears is explained using this model.

Two fluid model

General equation : $m\dfrac{\partial \vec{\mathbf{V}}}{\partial t} = q(\vec{\boldsymbol{E}} + \vec{\mathbf{v}} \times \vec{\boldsymbol{B}}) - \nu m \vec{\mathbf{v}}$

Two fluids (super current and normal current)

$$\vec{J} = \overrightarrow{J_S} + \overrightarrow{J_n}\ ,\ \overrightarrow{J_S} = n_S q_S \vec{V}_S\ ,\ \overrightarrow{J_n} = n_n q_n \vec{V}_n$$

Neglected the Lorentz term $\vec{\mathbf{v}} \times \vec{\boldsymbol{B}}$ because $\vec{\mathbf{v}}$ issmall.

$$m_S \frac{\partial \overrightarrow{V_S}}{\partial t} = q_S \vec{\boldsymbol{E}}\ ,\ m_S = 2m\ (Cooper\ pair), q_S = -2e$$

$$m_n \frac{\partial \overrightarrow{V_n}}{\partial t} = q_n \vec{\boldsymbol{E}},\ m_n = m\ (nornal\ electron), q_n = -e$$

Two Fluid Model

$$\vec{\boldsymbol{E}} = \overrightarrow{\boldsymbol{E}_0} \cdot \exp(i\omega t) \Rightarrow \overrightarrow{J_s} = \frac{n_s {q_s}^2}{i\omega m_s}\vec{E}, \quad \overrightarrow{J_n} = \frac{n_n e^2}{i(\omega - i\nu)}\vec{E}$$

$$\vec{J} = \left(\frac{n_s {q_s}^2}{i\omega m_s} + \frac{n_n e^2}{i(\omega - i\nu)m}\right)\vec{E}$$

$$\nu >> \omega \Rightarrow \vec{J} = \left(\frac{n_n e^2}{\nu m} - i\frac{n_s {q_s}^2}{\omega m_s}\right)\vec{E}$$

$$\vec{E} = (\sigma_n - i\sigma_s)\vec{E} = \sigma\vec{E}$$

$$\sigma \equiv \sigma_n - i\sigma_s$$

Surface Impedance for Superconductivity

$$Z = (1-i)\sqrt{\frac{\mu\omega}{2\sigma}} = (1-i)\sqrt{\frac{\mu\omega}{2(\sigma_n - i\sigma_s)}}$$

$$= (1-i)\sqrt{\frac{\mu\omega}{2}} \cdot \frac{1}{\sqrt{-i\sigma_s(1-\sigma_n / i\sigma_s)}}$$

$$\approx (1-i)\sqrt{\frac{\mu\omega}{2}} \cdot \frac{(1+\frac{\sigma_n}{i2\sigma_s})}{\sqrt{-i\sigma_s}}$$

$$= \sqrt{\mu\omega}\left(\frac{\sigma_n}{2\sigma_s^{3/2}} + i\frac{1}{\sigma_s^{1/2}}\right) \qquad \because \sqrt{-i} = \sqrt{e^{-\frac{\pi}{2}}} = \frac{(1-i)}{\sqrt{2}}$$

$$\mathrm{Z} = \frac{\mu\omega\lambda^3}{\delta^2} + i\mu\omega\lambda = \frac{\mu^2\omega^2\lambda^3\sigma_n}{2} + i\omega\mu\lambda$$

Here, $\lambda = \sqrt{\frac{m_s}{n_s q_s^2 \mu}} = c\sqrt{\frac{2m}{4n_s\mu e^2}} = c\sqrt{\frac{m}{2n_s\mu e^2}}$

SRF Surface Resistance

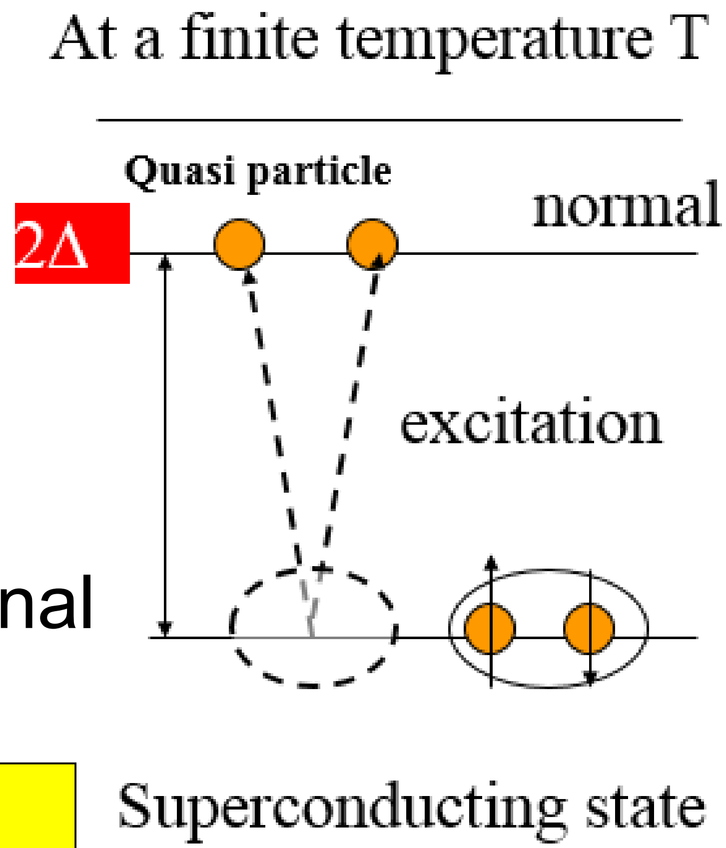
V_F : velocity at the Fermi surface, l: mean free path

$$\sigma_n = \frac{n_n \cdot e^2 \cdot l}{m \cdot v_F} = \frac{e^2 \cdot l}{m \cdot v_F} \cdot n_s(T=0) \cdot e^{-\frac{\Delta}{k_B \cdot T}}$$

$$R_S = \frac{1}{2} \cdot (2\pi)^2 \cdot \mu^2 \cdot f^2 \cdot \lambda_L^3 \cdot l \cdot \frac{n_s(0)}{m v_F} \cdot e^{-\frac{\Delta}{k_B T}}$$

$$= A \cdot f^2 \cdot e^{-\frac{\Delta}{k_B T}}$$

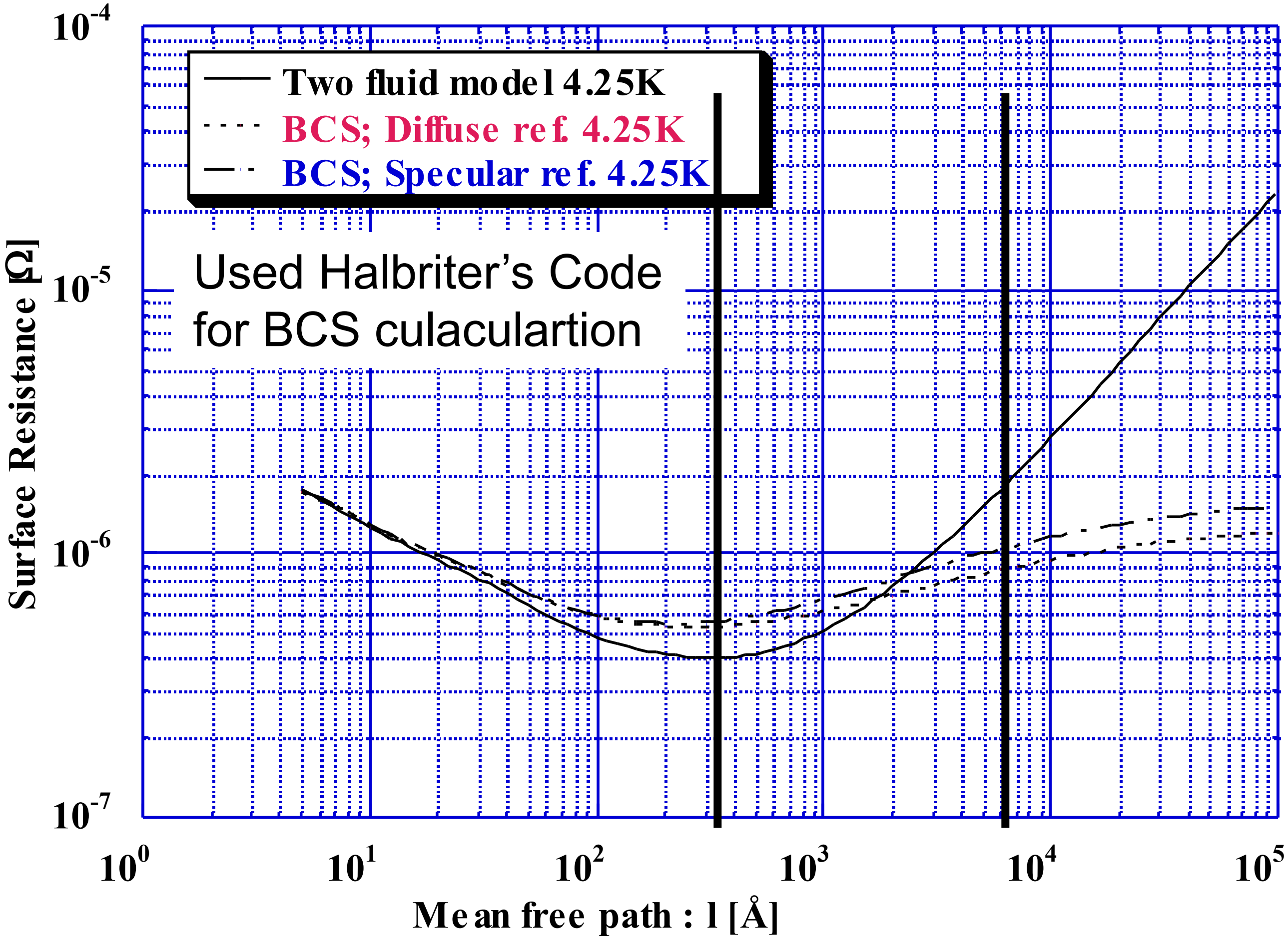
SRF resistance increases proportional to f^2.



BCS Theory

R_{BCS}=A(λ, ξ, l, T_C)$\cdot\frac{f^2}{T} \cdot \exp(-\frac{\Delta}{k_B T})$

Calculation of BCS Surface Resistance



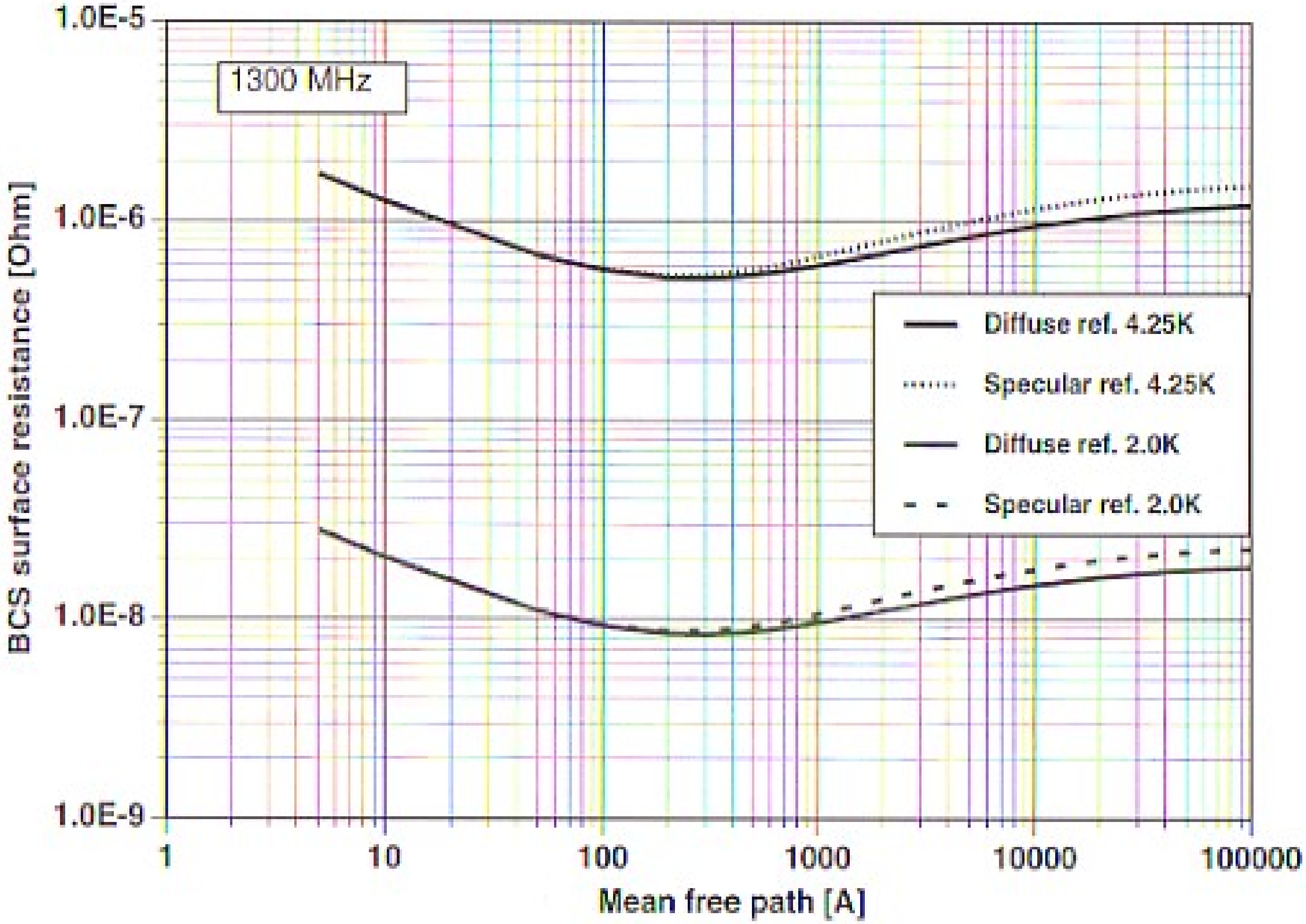
Strange behavior on Mean free path !!

R_{BCS} takes minimum value around l =200 ~ 300 Å

l(Å) ~ 20·RRR

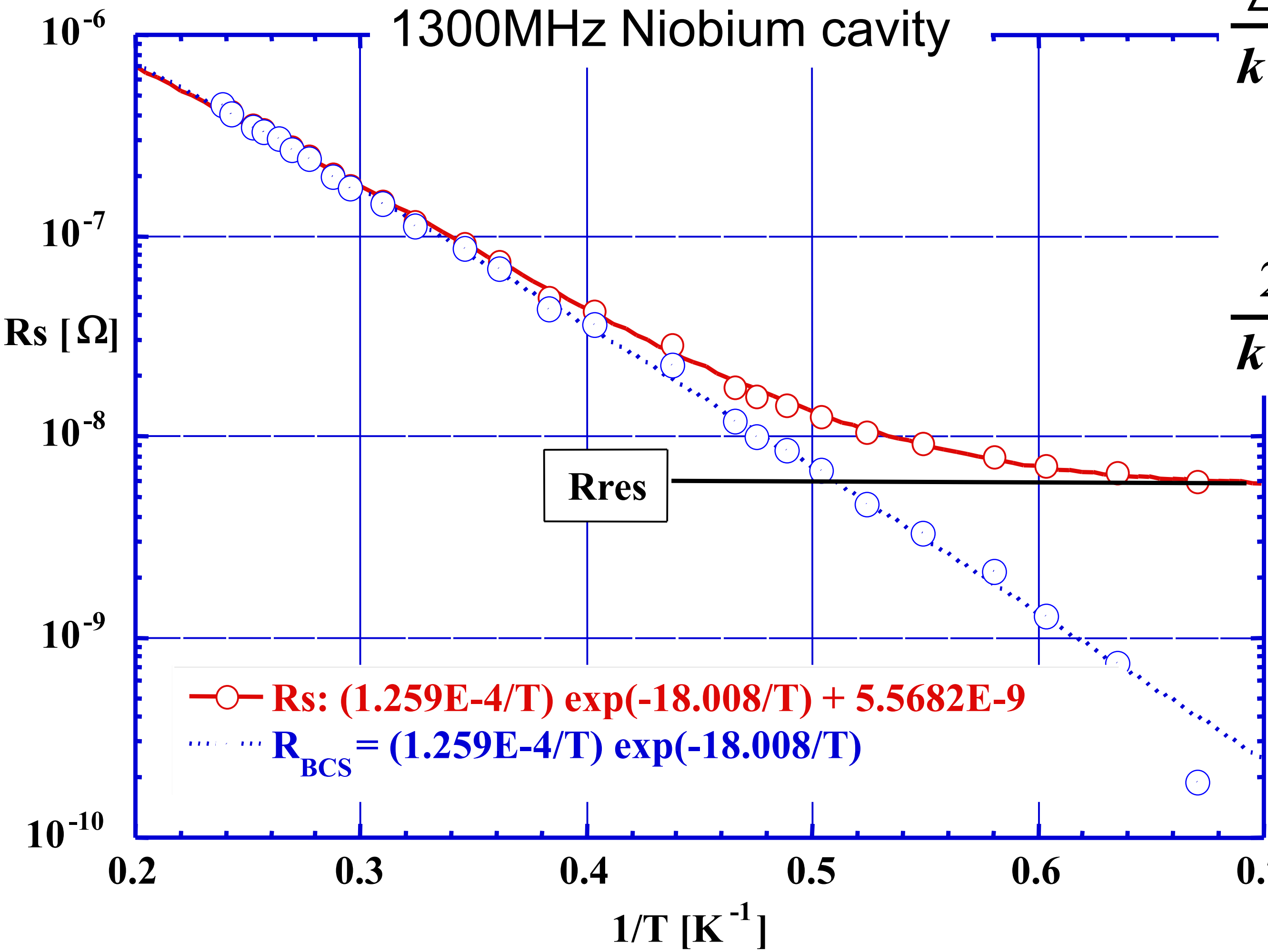
$$\lambda_L(l) = \lambda_L(l=\infty)\cdot\sqrt{1+\frac{\xi_o}{l}},\quad R_S(TF\ model) \propto (1+\frac{\xi_o}{l})^{\frac{3}{2}}\cdot l,\quad l \ll 1, R_S \to \frac{\xi_o^{\frac{3}{2}}}{\sqrt{l}},\quad l \gg 1, R_s \to l$$

BSC Surface Resistance with Niobium (1300 MHz)



$R_{BCS} \sim 8n\Omega$ @ 2K, 1300MHz

Measurement Result



$$\frac{\Delta}{k_B} = 18.008 \Rightarrow \frac{2\Delta}{k_B T_c} = \frac{2 \cdot 18.008}{9.25} = 3.89$$

$$\frac{2\Delta}{k_B T_c} = 3.52 \quad \textbf{(BCS theory)}$$

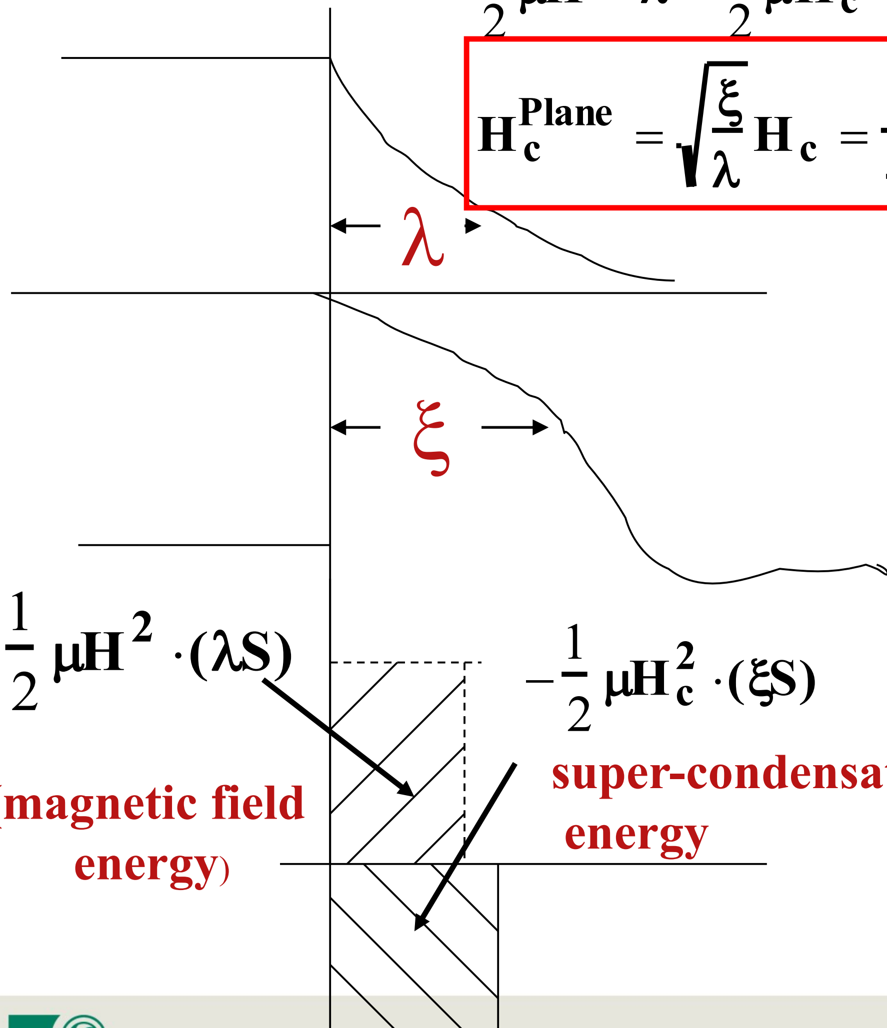
Measured $R_{BCS} \sim 8n\Omega$,
Measurement result fits well the BCS calculation.
Copper cavity 9.24 mΩ, SRF niobium cavity is 1.155×10^6 times small !!

5.6 Fundamental limitation

Vortex plane nucleation

$$\frac{1}{2}\mu H^2\cdot\lambda-\frac{1}{2}\mu H_c^2\cdot\xi=0$$

$$\boxed{H_c^{Plane}=\sqrt{\frac{\xi}{\lambda}}H_c=\frac{H_c}{\sqrt{\kappa}}}$$



$$\frac{1}{2}\mu H^2\cdot(\lambda S)$$

(magnetic field energy)

$$-\frac{1}{2}\mu H_c^2\cdot(\xi S)$$

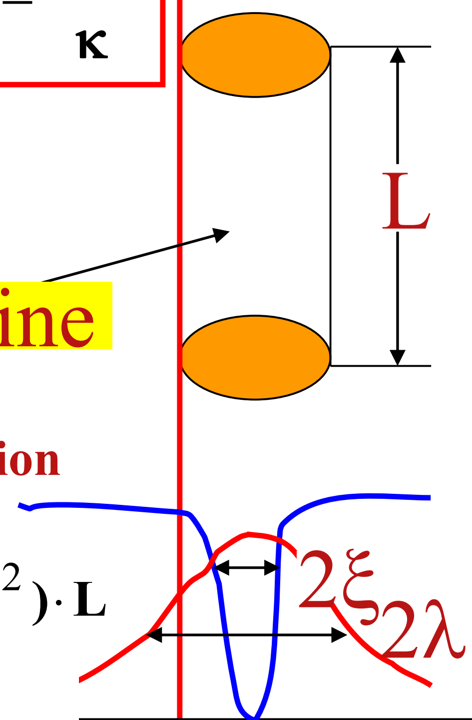
super-condensation energy

Vortex line nucleation

$$\frac{1}{2}\mu H^2\lambda^2-\frac{1}{2}\mu H_c^2\xi^2=0$$

$$\boxed{H_c^{Line}=\frac{\xi}{\lambda}H_c=\frac{H_c}{\kappa}}$$

Vortex line



super-condensation energy

$$-\frac{1}{2}\mu H_c^2(\pi\xi^2)\cdot L$$

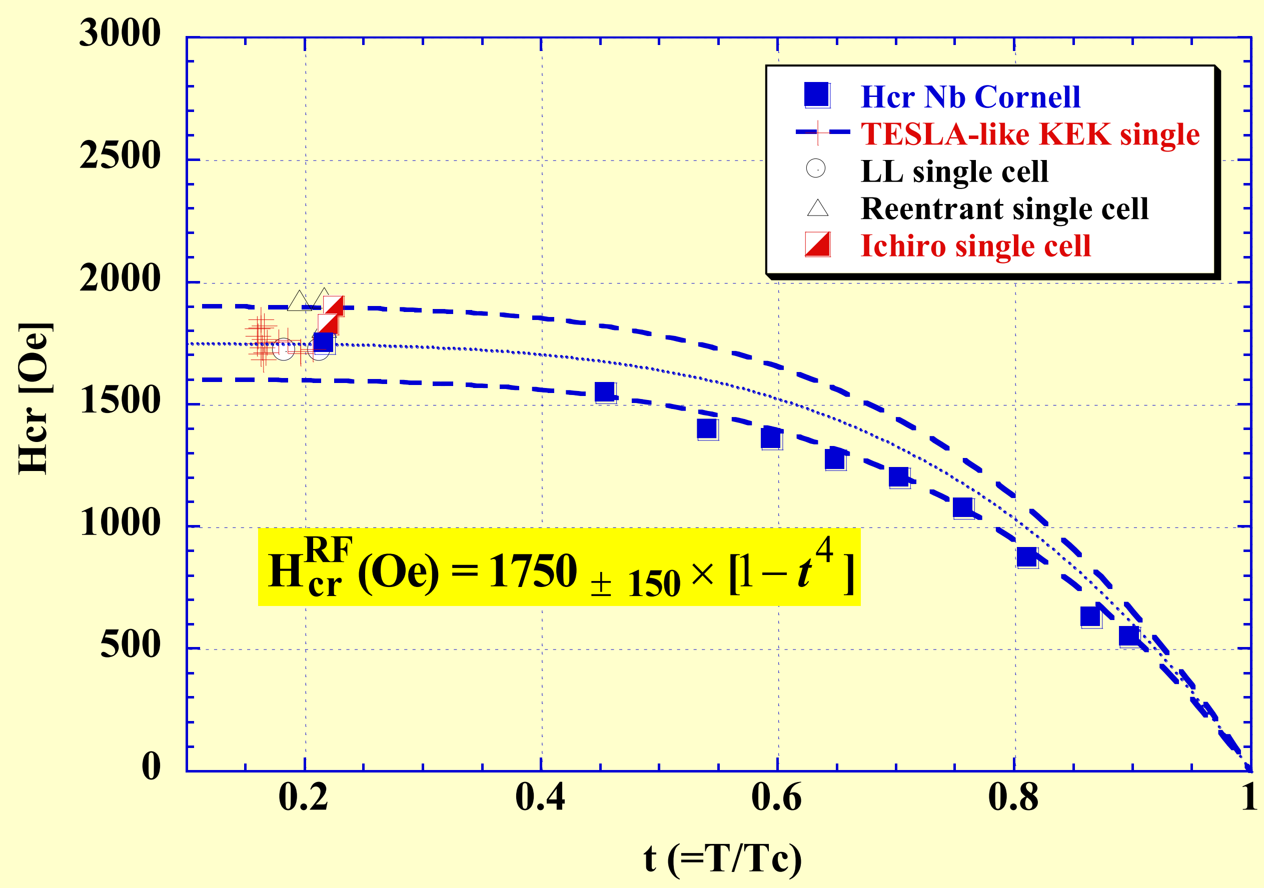
magnetic field energy

$$\frac{1}{2}\mu H^2(\pi\lambda)^2\cdot L$$

Temperature Dependence of Critical Field by of Vortex Line Model

Vortex line model can produce the temperature dependence but the absolute value is about $\sqrt{2}$ smaller.

$$H_c^{RF} = \frac{H_c(T)}{k(T)} = \frac{H_c(0)}{k(0)} \cdot \left[1 - \left(\frac{T}{T_c}\right)^4\right]$$



Measurement value

$$\frac{H_c(0)}{k(0)} = \frac{1950}{1.5} \sim \frac{2100}{1.6}$$

= 1300

Very Recent Result at Cornell
Superheating model

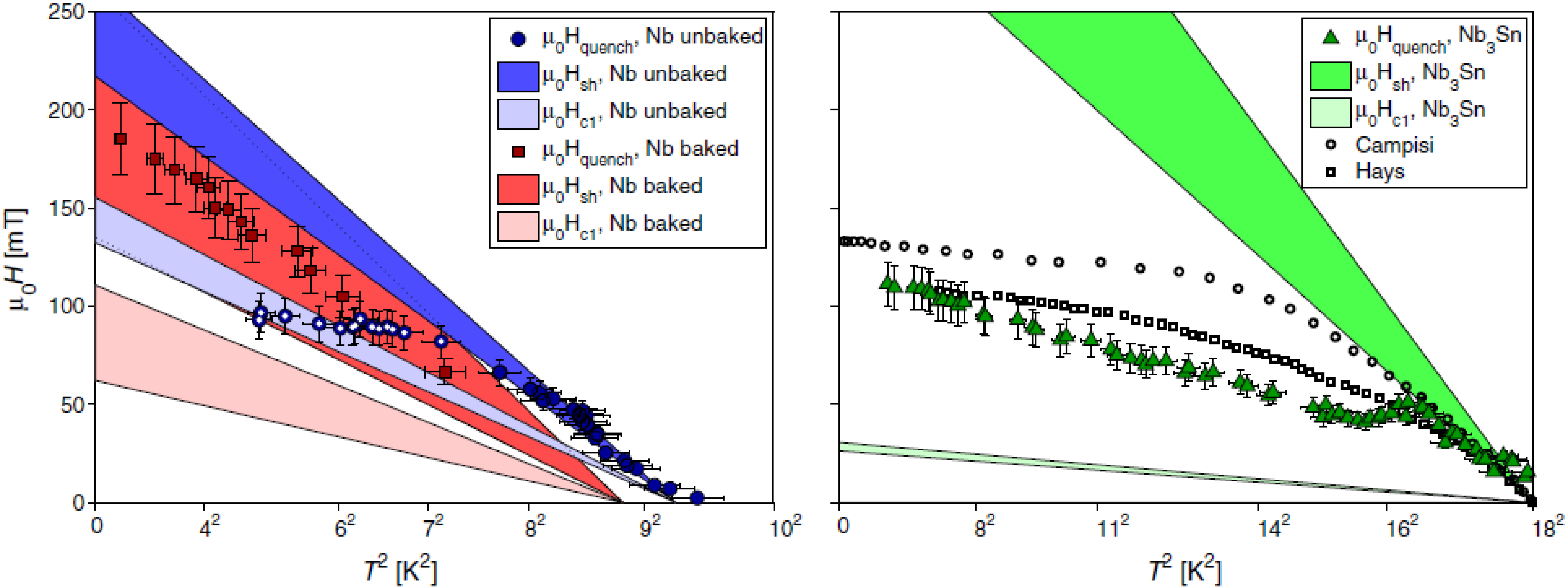


FIG. 1 (color online). Measured quench field of Nb (left) and Nb_3Sn (right) superconducting cavities under high power rf pulses as a function of temperature. For niobium, the cavity was tested both with bake to prevent limitation from HFQS and without it; the measurements that appear to be affected by HFQS are marked with white points at their centers. The dark and light shaded bands show the approximate temperature dependence of H_{sh} and H_{c1} extracted from CW measurements.

Suitable Superconductor for SRF Application

- Fundamental limitation point of view
  - Higher Tc
  - Larger Hc
  - Smaller GL κ parameter
- Fabrication point of view
  - Single phase
  - Easy forming

Materials	Tc [K]	Hc, Hc1 [Gauss]	Type	Fabrication
Pb	7.2	803	I	Electroplating
Nb	9.25	1900, 1700	II	Deep drawing, film
Nb3Sn	18.2	5350, 300	II	Film
MgB2	39	4290, 300	II	Film

Homework

Homework 1

In this lecture, why SRF application has surface resistance was explained different from DC application.

However, please make your summary note about this by yourself.